Singapore Mathematical Society

Singapore Mathematical Olympiad (SMO) 2009

(Senior Section, Round 2)

Saturday, 27 June 2009 **0930-1230**

INSTRUCTIONS TO CONTESTANTS

1. *Answer ALL 5 questions.*
2. *Show all the steps in your working.*
3. *Each question carries 10 mark.*
4. *No calculators are allowed.*

1. Let M and N be points on sides AB and AC of triangle ABC respectively. If

$$\frac{BM}{MA}+\frac{CN}{NA}=1,$$

show that MN passes through the centroid of ABC.

2. Find all pairs of positive integers n, m that satisfy the equation $3\cdot 2^m+1=n^2$.

3. L et A be an n-element subset of $\{1,2,\ldots,2009\}$ with the property that the difference between any two numbers in A is not a prime number. Find the largest possible value of n. Find a set with this number of elements. (Note: 1 is not a prime number.)

4. Let $a,b,c>0$ such that $a+b+c=1$. Show that if if $x_1,x_2,\ldots,x_5$ are positive real numbers such that $x_1x_2\ldots x_5=1$, then

$$(ax_1^2+bx_1+c)(ax_2^2+bx_2+c)\cdots(ax_5^2+bx_5+c)\geq 1.$$

5. In an archery competition, there are 30 contestants. The target is divided in two zones. A hit at zone 1 is awarded 10 points while a hit at zone 2 is awarded 5 points. No point is awarded for a miss. Each contestant shoots 16 arrows. At the end of the competition statistics show that more than 50% of the arrows hit zone 2. The number of arrows that hit zone 1 and miss the target are equal. Prove that there are two contestants with the same score.