Singapore Mathematical Society

Singapore Mathematical Olympiad (SMO) 2009

(Junior Section, Round 2)

1. In $\triangle ABC$, $\angle A = 2\angle B$. Let a, b, c be the lengths of its sides BC, CA, AB, respectively. Prove that

$$a^2 = b(b+c).$$

2. The set of 2000-digit integers are divided into two sets: the set M consisting all integers each of which can be represented as the product of two 1000-digit integers, and the set N which contains the other integers. Which of the sets M and N contains more elements?

3. Suppose $\overline{a_1 a_2 \ldots a_{2009}}$ is a 2009-digit integer such that for each $i = 1, 2, \ldots, 2007$, the 2-digit integer $\overline{a_i a_{i+1}}$ contains 3 distinct prime factors. Find a_{2008}. (Note: $\overline{xyz\ldots}$ denotes an integer whose digits are $x, y, z, \ldots$.)

4. Let S be the set of integers that can be written in the form $50m + 3n$ where m and n are non-negative integers. For example 3, 50, 53 are all in S. Find the sum of all positive integers not in S.

5. Let a, b be positive real numbers satisfying $a + b = 1$. Show that if $x_1, x_2, \ldots, x_5$ are positive real numbers such that $x_1 x_2 \ldots x_5 = 1$, then

$$(ax_1 + b)(ax_2 + b) \cdots (ax_5 + b) \geq 1.$$