SMO Junior 2024 Rd.2

1. Let ABC be an isosceles right-angled triangle of area 1. Find the length of the shortest segment that divides the triangle into 2 parts of equal area.

2. Let $ABCD$ be a parallelogram and the points E, F are in the exterior. If triangles BCF and DEC are similar, i.e. $\Delta BCF \sim \Delta DEC$, prove that triangle AEF is similar to these two triangles.

3. Seven triangles of area 7 lie inside a square of area 27. Prove that among the 7 triangles there are 2 that intersect in a region of area not less than 1.

4. Suppose for some positive integer n, the numbers 2^n and 5^n have equal first digit. What are the possible values of this first digit?

5. Find all integer solutions of the equation

$$y^2 + 2y = x^4 + 20x^3 + 104x^2 + 40x + 2003.$$