

**Singapore Mathematical Society**  
**Singapore Mathematical Olympiad (SMO) 2007**  
(Senior Section, Round 2)

Saturday, 30 June 2007

0930-1230

**Instructions to contestants**

1. Answer ALL 5 questions.
2. Show all the steps in your working.
3. Each question carries 10 mark.
4. No calculators are allowed.

1. It is given that  $x, y, z$  are 3 real numbers such that

$$\frac{x-y}{2+xy} + \frac{y-z}{2+yz} + \frac{z-x}{2+zx} = 0.$$

Is it true that at least two of the three numbers must be equal? Justify your answer.

2. For any positive integer  $n$ , let  $f(n)$  denote the  $n$ th positive nonsquare integer, i.e.,  $f(1) = 2, f(2) = 3, f(3) = 5, f(4) = 6$ , etc. Prove that

$$f(n) = n + \{\sqrt{n}\}$$

where  $\{x\}$  denotes the integer closest to  $x$ . (For example,  $\{\sqrt{1}\} = 1, \{\sqrt{2}\} = 1, \{\sqrt{3}\} = 2, \{\sqrt{4}\} = 2$ .)

3. In the equilateral triangle  $ABC$ ,  $M, N$  are the midpoints of the sides  $AB, AC$ , respectively. The line  $MN$  intersects the circumcircle of  $\triangle ABC$  at  $K$  and  $L$  and the lines  $CK$  and  $CL$  meet the line  $AB$  at  $P$  and  $Q$ , respectively. Prove that  $PA^2 \cdot QB = QA^2 \cdot PB$ .
4. Thirty two pairs of identical twins are lined up in an  $8 \times 8$  formation. Prove that it is possible to choose 32 persons, one from each pair of twins, so that there is at least one chosen person in each row and in each column.
5. Find the maximum and minimum of  $x + y$  such that

$$x + y = \sqrt{2x-1} + \sqrt{4y+3}.$$