Singapore Mathematical Society

Singapore Mathematical Olympiad (SMO) 2007

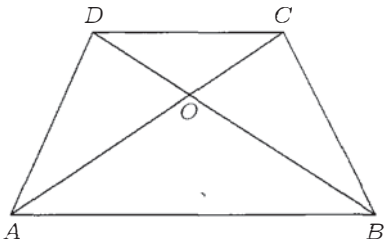
(Junior Section, Round 2)

Saturday, 30 June 2007 **0930-1230**

Instructions to contestants

1. *Answer ALL 5 questions.*
2. *Show all the steps in your working.*
3. *Each question carries 10 mark.*
4. *No calculators are allowed.*

1. In the f llowing figure, $AB \parallel DC$, $AB = b$, $CD = a$ and $a < b$. Let S be the area of the trapezium $ABCD$. Suppose the area of $\triangle BOC$ is $2S/9$. Find the value of a/b.



2. Equilateral triangles ABE and BCF are erected externally on the sides AB and BC of a parallelogram $ABCD$. Prove that $\triangle DEF$ is equilateral.

3. Let n be a positive integer and d be the greatest common divisor of $n^2 + 1$ and $(n+1)^2 + 1$. Find all the possible values of d. Justify your answer.

4. The difference between the product and the sum of two different integers is equal to the sum of their GCD (greatest common divisor) and LCM (least common multiple). Find all these pairs of numbers. Justify your answer.

5. For any positive integer n, let $f(n)$ denote the nth positive nonsquare integer, i.e., $f(1) = 2$, $f(2) = 3$, $f(3) = 5$, $f(4) = 6$, etc. Prove that

$$f(n) = n + \{\sqrt{n}\}$$

where $\{x\}$ denotes the integer closest to x. (For example, $\{\sqrt{1}\} = 1$, $\{\sqrt{2}\} = 1$, $\{\sqrt{3}\} = 2$, $\{\sqrt{4}\} = 2$.)