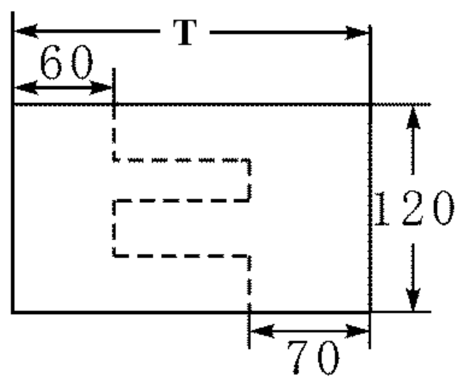
2015 World Mathematics Team Championship
Junior Level Relay Round 1
Solutions

Problems	1A	1B	2A	2B	3A	3B
Answers	199	1292	8	128	5	20

1A. Define a & $b = a \times a - b \times b$. Find 100 & 99.

Solution: 199. 100 & 99 = $(100\times100) - (99\times99) = 100^2 - 99^2 = (100+99)\times(100-99) = 199$.

1B. Let T = TNYWR (The Number You Will Receive). Given a piece of rectangular paper of length T and width 120. Cut this piece of paper into two parts along the dotted line as shown in the figure below. Find the sum of the perimeters of both parts.

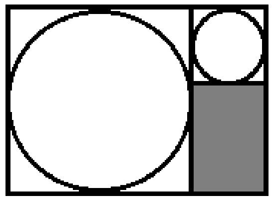


Solution: 1292. Combined perimeters $= (2\times T) + (2\times120) + (2\times120) + 2\times3\times(T-60-70)$
$= 2T + 240 + 240 + 6T - 780$
$= 8T - 300$
$= 8\times199 - 300$ since $T = 199$
$= 1292$.

2A. If both 2–digit numbers $\overline{ab}$ and $\overline{ba}$ ($a \neq b$) are prime numbers, we would call them "WMTC numbers". How many possible 2–digit WMTC numbers are there?

Solution: 8. 13, 31, 17, 71, 37, 73, 79, 97 are the only 2–digit WMTC numbers. Noted that 19 and 91 are not since $91 = 7\times13$ not a prime.

2B. Let T = TNYWR (The Number You Will Receive). As shown in the figure below, cut out two circles from a piece of rectangular piece of paper so that the area of the large circle is 9 times as large as the area of the small circle. If the diameter of the small circle is T, find the area of the shaded (rectangular) region.



Solution: 128. Let r be the radius of the large circle. Because the diameter of the small circle is T, so $\pi r^2 = 9\pi(T/2)^2$ which means $r = 3T/2$. Hence the dimensions of this rectangle are $(2r+T) \times (2r)$. So, the area of the shaded region is (Area of outside rectangle) – (Area of square around the area circle) – (Area of the square around the small circle) = $(2r+T) \times (2r) - (2r)^2 - T^2$ or $2T^2$ or $2(8)^2 = 128$.

3A. If the remainder is 3 when a is divided by 11 and the remainder is 5 when b is divided by 11, find the remainder when $4a+3b$ is divided by 11.

Solution: 5. $a \equiv 3 \pmod{11}$ and $b \equiv 5 \pmod{11}$. Therefore $4a+3b \equiv 4(3)+3(5) \pmod{11} \equiv 27 \pmod{11} \equiv 5 \pmod{11}$.

3B. Let T = TNYWR (The Number You Will Receive). A shop needs to produce a number of pairs of gloves. Suppose A can finish this job by himself in 30 hours, B can finish this job by himself in 40 hours, and C can finish this job by herself in 60 hours. If A, B, and C started to work at 8 o'clock and A completed T more pairs of gloves than B at 10 o'clock, how many more pairs of gloves did A complete more than C at 12 o'clock?

Solution: 20. Suppose this shop needs to produce x pairs of gloves. In one hour, A can make $1/30x$ pairs of gloves, B can make $1/40x$ pairs, and C can make $1/60x$ pairs. In two hours (from 8:00 to 10:00), A can do $2/30x$ pairs and B can make $2/40x$ pairs. So, $2/30x - 2/40x = 1/60x = T$ or $x = 60T$. In 4 hours (from 8:00 to 12:00), A can make $4/30x$ and C can make $4/60x$. Therefore, in 4 hours, A make make $4/30x - 4//60x = 4/60x = 4T = 4(5) = 20$ more pairs of shoes than C.