2015 World Mathematics Team Championship
Intermediate Level Relay Round
Solutions

Problems	1A	1B	2A	2B	3A	3B
Answers	2	$2\sqrt{3}$	540	395	89	109

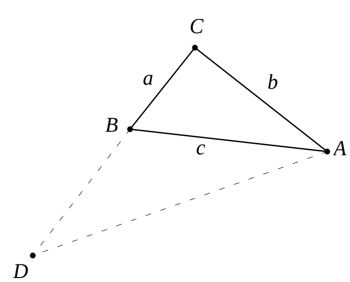
1A. If $\frac{a}{b}=\frac{c}{d}=2$, find $\frac{3a-c}{3b-d}$.

Solution: 2. Since $\frac{a}{b}=\frac{c}{d}=2$, so $a = 2b$ and $c = 2d$. Substitute these into

$$\frac{3a-c}{3b-d} = \frac{3\times 2b-2d}{3b-d} = 2.$$

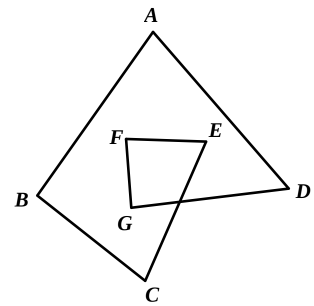
1B. Let $T =$ TNYWR (The Number You Will Receive). Let a, b, and c be the sides that are opposite to angles A, B, and C, respectively, of ΔABC. If $\frac{a}{b}=\frac{a+b}{a+b+c}$, $\angle A=30°$, and $a = T$, find the area of ΔABC.

Solution: $2\sqrt{3}$. As shown in the figure on the right, extend CB to D so that $BD = AB = c$. Connect AD.

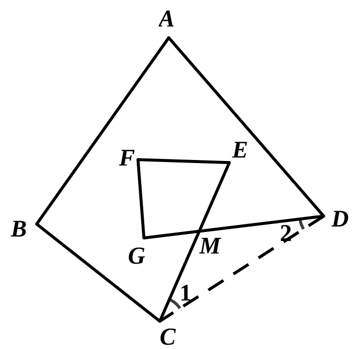


Since $\frac{a}{b}=\frac{a+b}{a+b+c}$, so $\frac{BC}{AC}=\frac{a}{b}=\frac{b}{a+c}=\frac{AC}{CD}$. Since $\angle C$ is the common angle for ΔABC and ΔDAC, so $\Delta ABC \backsim \Delta DAC$ and that means $\angle BAC = \angle D$. Also, $\angle BAD = \angle D$ and $\angle BAC=30°$,
so $\angle ABC = \angle D + \angle BAD = 2\angle D = 2\angle BAC = 60°$
and $\angle C = 90°$ which means ΔABC is a right triangle.
Since $T = 2 = a$, so $b = 2\sqrt{3}$ and the area of right triangle ΔABC is
$S_{\Delta ABC} = \frac{1}{2}ab = \frac{1}{2}\times 2\times 2\sqrt{3} = 2\sqrt{3}$.

2A. As shown in the figure below, let $\angle A+\angle B+\angle C+\angle D+\angle E+\angle F+\angle G=x^{\circ}$. Find x.



Solution: 540°. As shown in the figure below, let M be the intersection point of CE and DG. Connect CD.



Because $\angle A+\angle B+\angle BCD+\angle CDA=360^{\circ}$, $\angle A+\angle B+\angle BCE+\angle 1+\angle 2+\angle GDA=360^{\circ}$. ①

Also, $\angle E+\angle F+\angle G+\angle GME=360^{\circ}$, ② and

$\angle 1+\angle 2+\angle GME=\angle 1+\angle 2+\angle CMD=180^{\circ}$. ③

Hence ①+②−③ =

$(\angle A+\angle B+\angle BCE+\angle 1+\angle 2+\angle GDA)+(\angle E+\angle F+\angle G+\angle GME)-(\angle 1+\angle 2+\angle GME)$

$=\angle A+\angle B+\angle BCE+\angle GDA+\angle E+\angle F+\angle G=360^{\circ}+360^{\circ}-180^{\circ}=540^{\circ}$.

2B. Let $T=$ TNYWR (The Number You Will Receive). Suppose x and y are integers that satisfy the set of equations $\begin{cases} x+3y^2+2xy=18, & ① \\ y+3x^2+4xy=6, & ② \end{cases}$.

Find the value for $T\times(x+y)+2015$.

Solution: 395. From ①+②, we have $x+3y^2+2xy+y+3x^2+4xy=24$ or

$x+y+3(x^2+y^2)+6xy=(x+y)+3(x^2+y^2+2xy)=(x+y)+3(x+y)^2=24$ or

$[3(x+y)-8]\cdot[(x+y)+3]=0$ or $(x+y)=\dfrac{8}{3}$ or -3.

Since x and y are integers, $x+y$ is also integer and $(x+y)$ must be –3. Therefore,

$T\times(x+y)+2015 = 540\times(-3)+2015 = 395$.

3A. The cube of a natural number can be written as the sum of two or more consecutive odd numbers. For examples, $2^3 = 3+5$, $3^3 = 7+9+11$, and $4^3 = 13+15+17+19$. If 9^3 is written as the sum of two or more consecutive odd numbers, what is the largest odd number in this sum?

Solution: 89. From observation, 2^3 can be written as sum of two consecutive odd numbers of 3 and 5, 3^3 can be written as sum of three consecutive odd numbers of 7, 9, and 11, and 4^3 can be written as sum of four consecutive numbers of 13, 15, 17, and 19. So, we should be able to write 9^3 as sum of 9 consecutive odd numbers with the first odd number being the (2+3+4+5+6+7+8+1)th = 36th odd number starting with 3 and the largest or the last odd number being the (35+9)th = 44th odd numbers starting with 3. Therefore, the largest odd number in the 9 odd numbers that sum up to 9^3 is $2\times 44+1=89$.

3B. Let T = TNYWR (The Number You Will Receive). Suppose p and q are non–zero natural numbers and that $p < q$. If $\frac{p}{q} = 0.18\cdots$ and $q = 110$, find $(p + T)$.

Solution: 109. Since $\frac{p}{q} = 0.18\cdots$, $0.18 < \frac{p}{q} < 0.19$ or $\frac{18}{100} < \frac{p}{q} < \frac{19}{100}$ or

$\frac{18}{100} < \frac{p}{110} < \frac{19}{100}$ or $19.8 < p < 20.9$. p is a natural number so $p = 20$.

Therefore, $p + T = 20 + 89 = 109$.