Solutions – Upper Primary Division

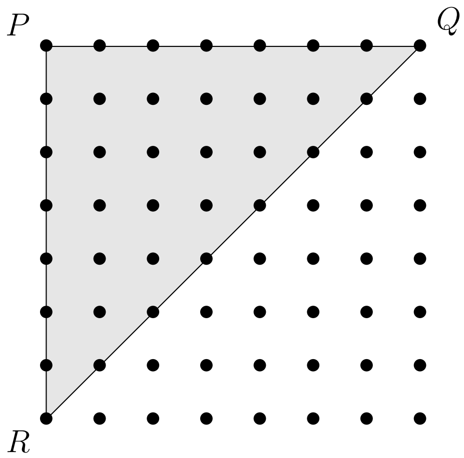
1. (Also MP2)
The numbers in order are 555, 556, 565, 566, 655,

hence (D).

2. (Also MP6)
One pizza will have 4 quarters, so two pizzas will have $2 \times 4 = 8$ quarters,

hence (D).

3.



The number of interior lattice points is $1 + 2 + 3 + 4 + 5 = 15$,

hence (C).

4. $0.3 + 0.4 = 0.7$,

hence (B).

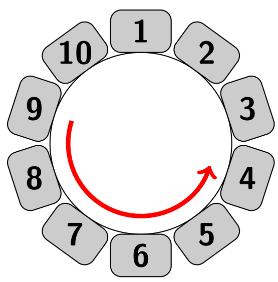
5. (Also MP10)
Alternative 1
In cents, $500 \div 80 = 6r20$ so that he buys 6 chocolates and has 20 cents left,

hence (C).

Alternative 2
Multiples of 80 are $80, 160, 240, 320, 400, 480, 560$. From this, he can afford 6 chocolates but not 7,

hence (C).

6. (Also MP9)
The opposite chair is both 5 places forward and 5 places back. Five places back from chair 9 is chair 4,



hence (D).

7. In order, 𝅝 𝅗𝅥 ♩ ♪ ♪ is worth $4 + 2 + 1 + \frac{2}{2} = 8$ beats,

hence (E).

8. (Also MP13)
She either has a 50c coin or not.
If she has a 50c coin, then she has one other 10c coin: $50 + 10 = 60$.
If she has no 50c coins, then she either has 0, 1, 2 or 3 20c coins:

$$20 + 20 + 20 = 60 \qquad 20 + 20 + 10 + 10 = 60$$
$$20 + 10 + 10 + 10 + 10 = 60 \qquad 10 + 10 + 10 + 10 + 10 + 10 = 60$$

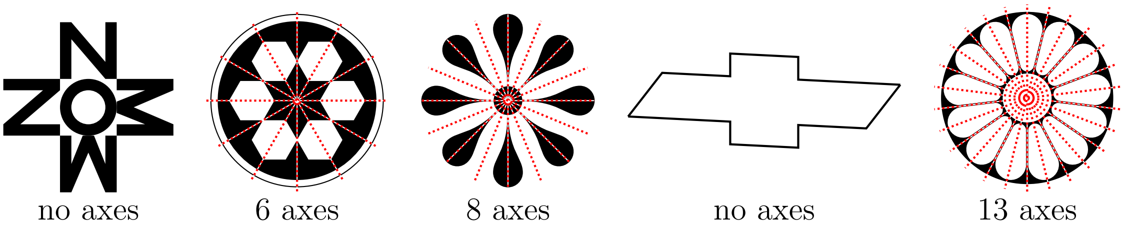
In all, there are 5 possibilities,

hence (D).

9. (A) holds $\frac{1}{3}$ of 3000 mL which is 1000 mL.
(B) holds $\frac{3}{4}$ of 1000 mL which is 750 mL.
(C) holds $\frac{1}{2}$ of 1000 mL which is 500 mL.
(D) holds $\frac{1}{3}$ of 750 mL which is 250 mL.
(E) holds $\frac{1}{4}$ of 2000 mL which is 500 mL.
Of these 1000 mL is the greatest,

hence (A).

10. Here are the axes of symmetry of each shape:



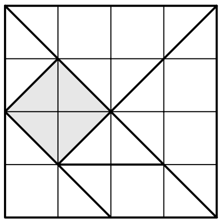
hence (E).

11. (Also MP15)
Starting from the outer end of the spiral (the loop on the rope) the dark and light sections are longest, and the light sections are of similar length to the dark sections. As you move towards the other end of the rope, both dark and light sections get shorter. Only rope (A) shows this,

hence (A).

12. (Also MP18)

The large square must have side 8 cm. Then all of the corners of the pieces in the tangram lie on a grid of 2 cm × 2 cm squares.



The shaded square has the same area as 2 of the grid squares, or $2 \times (2\,\text{cm} \times 2\,\text{cm}) = 8\,\text{cm}^2$,

hence (D).

13. *Alternative 1*

Since $8 \times 25 = 200$, he needs to make 8 batches. This requires $8 \times 2\frac{1}{2} = 20$ packets of chocolate chips,

hence (A).

Alternative 2

Each packet of chocolate chips is enough for 10 biscuits. So for 200 biscuits, 20 packets are required,

hence (A).

14. Two even numbers add to even. For example, $2 + 4 = 6$. So, not (A).

The difference between two odds is always even. For example, $5 - 1 = 4$. So, not (B).

The sum of two odd numbers is always even. For example, $5 + 1 = 6$. So, not (C).

Adding 3 odd numbers is the same as odd plus even, which is odd. For example, $3 + 5 + 9 = 8 + 9 = 17$. So (D) is true.

Two odd numbers multiplied is always odd. For example, $3 \times 7 = 21$. So, not (E),

hence (D).

15. The side of the outer square is $36 \div 4 = 9\,\text{cm}$ and the side of the inner square is $20 \div 4 = 5\,\text{cm}$.

The difference is 4 cm, which is 2 cm on each side. So each rectangle is $7\,\text{cm} \times 2\,\text{cm}$, with perimeter 18 cm,

hence (E).

16. *Alternative 1*

The possibilities are

First number	Possibilities	Count
2	2134, 2143	2
3	3124, 3142, 3214, 3241	4
4	4123, 4132, 4213, 4231, 4312, 4321	6
		12

hence (B).

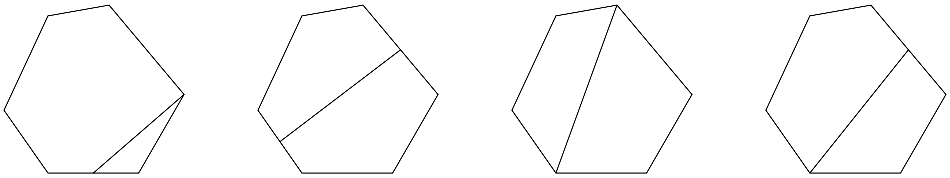
Alternative 2

Ignoring whether the first number is larger or smaller than the second, the number of combinations is $4 \times 3 \times 2 \times 1 = 24$.

Half of these will have the first number larger than the second, and half will be the other way around. So there are 12 combinations,

hence (B).

17. Each of (A), (B), (C) and (D) is possible:



However (E) is not possible. It has a total of 7 sides, whereas the hexagon has 6 sides and a cut increases the number of sides by at least 2, giving 8 or more sides,

hence (E).

18. As the sum $3 + 9 + 15 + 18 + 24 + 29 = 98$, we are looking for two groups of three numbers each with a sum close to 49. There are no three of the numbers adding to 49, so the difference must be 2 or more.

To get the difference as close as possible to 0, each sum will be as close to 49 as possible.

Noting that $3+18+29 = 50$ and $9+15+24 = 48$ gives $(3+18+29)-(9+15+24) = 2$,

hence (C).

19. Adding the upward-sloping diagonal, $4 + 7 + 10 + 13 = 34$ is the common total.

In the 4th column $13 + 12 + 1 = 26$ and $34 - 26 = 8$. In the 2nd row, $5 + 10 + 8 = 23$ and $34 - 23 = 11$. Place A and B in the third row as shown.

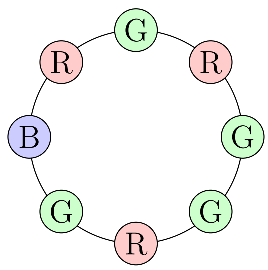
X			13
5	11	10	8
A	7	B	12
4			1

From the first column, $A + X = 34 - 9 = 25$. From the downward-sloping diagonal, $B + X = 34 - 12 = 22$. From the third row, $A + B = 34 - 19 = 15$.

Then $25 + 22 + 15 = 62$ counts each of A, B and X twice, so that $A + B + X = 31$ and $X = 31 - 15 = 16$,

hence (A).

20. (Also J18)



First try not to use a blue counter at all.

The counters can't be all red or all green, so start with a red counter at the bottom of the circle. By the first rule, the counters either side must be green.

Then, by the second rule, the counters opposite these green counters must be red. The top counter can now be coloured green.

The side counters cannot be red because they are adjacent to a red. However, only one of them can be green, so the other must be blue. This arrangement, as shown, satisfies the three rules.

Hence, the minimum number of blue counters is 1,

hence (B).

21. (Also MP23)
The discs are back in their original positions after 5 moves.

r		g		o		b		y		r
b	1	y	2	r	3	g	4	o	5	b
g	$\longrightarrow$	o	$\longrightarrow$	b	$\longrightarrow$	y	$\longrightarrow$	r	$\longrightarrow$	g
y		r		g		o		b		y
o		b		y		r		g		o

They will be in their original positions again after 10, 15 and 20 moves. After 1 more move, blue will be on the bottom,

hence (B).

22. (Also MP24)
Alternative 1
Replacing the leopard by another lion (of the same weight as the lion) would add 90 kg, and replacing the tiger by another lion would add 50 kg. Then 3 lions weigh $310 + 90 + 50 = 450$ kg and 1 lion weighs $450 \div 3 = 150$ kg,

hence (B).

Alternative 2
If the lion weighs 100 kg, then the leopard weighs 10 kg and the tiger 50 kg for a total of 160 kg. This is 150 kg too light. Adding $150 \div 3 = 50$ kg to each weight keeps the differences in weight the same. So the lion weighs 150 kg,

hence (B).

23. (Also J15, I15)
As the number of points per event is 6 and the total number of points gained is $8 + 11 + 5 = 24$, there must have been 4 events. As Betty has 11 points she must have 3 first places and one second place. Cathy could not have won an event, or her score would be 6 or more, so she must have just one second place. So Adrienne must have two second places,

hence (C).

24. Arrange the coins as follows:

	A	B	C
Jane	5c coins	10c coins	50c coins
Tom	10c coins	20c coins	5c coins

Suppose Jane has just one 50c coin, so that Tom has one 5c coin and Jane has 45c more than Tom in column C.

Tom and Jane have the same amount, so than in columns A and B, Tom has 45c more than Jane. But for each coin in columns A and B, Tom's coin is worth twice Jane's coin, so Tom has 90c and Jane has 45c. The fewest coins for this is 1 coin each in column A and 4 coins each in column B. Then they have 6 coins each.

If Jane has more than one 50c coin, the difference in columns A and B will be 90c or more, which requires more than 4 coins in these columns. Then they have more than 6 coins each.

So the smallest number of coins they can each have is 6, when Jane has $1 \times 50c + 4 \times 10c + 1 \times 5c$ and Tom has $1 \times 5c + 4 \times 20c + 1 \times 10c$,

hence (D).

25. *Alternative 1*

The final mixture will have 9 litres of cordial out of 18 litres.

The amount of the mixture from Jar A can vary from 0 litres to 18 litres, so we try increasing amounts from Jar A and calculate the amount of 100% cordial in the mixture.

Litres from Jar A	0	1	2	3	$\cdots$	
Litres from Jar B	18	17	16	15	$\cdots$	
Litres of cordial from Jar A	0	0.3	0.6	0.9		
Litres of cordial from Jar B	10.8	10.2	9.6	9.0		
Litres of cordial in mixture	10.8	10.5	10.2	9.9	$\cdots$	9.0

For every additional litre from Jar A there is 0.3 litres less cordial in the mixture, which is 0.6 additional litres from Jar B and 0.3 fewer litres from Jar A.

Following this pattern, there will be 9 litres of cordial when there are 6 litres from Jar A and 12 litres from Jar B,

hence (E).

Note: Checking this, Jar A contributes 1.8 litres of cordial and Jar B contributes 7.2 litres of cordial.

Alternative 2

Let the amount from Jar A be x, and the amount from Jar B be y. Then

$$x + y = 18 \qquad\qquad \text{so} \qquad x = 18 - y$$

$$0.3x + 0.6y = 0.5 \times 18 \qquad \text{so} \qquad 3x + 6y = 90 \qquad \text{and} \qquad x = 30 - 2y$$

Therefore $18 - y = 30 - 2y$, which has solution $y = 12$, and then $x = 6$,

hence (E).

26. *Alternative 1*

Adding $11 + 17 + 22 = 50$ includes every hat twice. For example, Qiang's hat is in both Rory's and Sophia's totals.

Therefore the total of all three hats is 25. Then the person whose total is 11 has $25 - 11 = 14$ on their hat. The other two hats are $25 - 17 = 8$ and $25 - 22 = 3$.

So the three numbers are $3, 8$ and 14,

hence (14).

Alternative 2

Let the numbers be a, b and c. Then $a + b = 11$, $b + c = 17$ and $a + c = 22$. Then $2a + 2b + 2c = 50$, therefore $a + b + c = 25$. This gives $a = 8, b = 3, c = 14$ so the largest number is 14,

hence (14).

27. *Alternative 1*

We can test factors by division, finding each factor's partner until we see a factor that has occurred as a partner already.

Factor	1	2	3	4	5	6	7	8	9	10
Partner	840	420	280	210	168	140	120	105	–	84

Factor	11	12	13	14	15	16	17	18	19	20
Partner	—	70	—	60	56	—	—	—	—	42

Factor	21	22	23	24	25	26	27	28	29	30
Partner	40	—	—	35	—	—	—	30	—	28

From this, there are 16 pairs of factors, giving 32 factors,

hence (32).

Alternative 2

Factorised into primes, $840 = 2^3 \times 3 \times 5 \times 7$. Then every factor of 840 can be found by multiplying together:

- a factor of 8 (1, 2, 4 or 8)
- a factor of 3 (1 or 3)
- a factor of 5 (1 or 5)
- a factor of 7 (1 or 7)

This means there are $4 \times 2 \times 2 \times 2 = 32$ possible factors of 840,

hence (32).

28. (Also MP29)

The first cube uses 12 matches, then each subsequent cube uses 8 matches. Since $2016 - 12 = 2004$ and $2004 \div 8 = 250\text{r}4$, there are $1 + 250 = 251$ cubes made, with 4 matches left over,

hence (251).

29. (Also J27, I24)

There are three cases for how the triangles are coloured:

(i) All three sides are the same colour, with 5 possibilities.

(ii) Two sides are the same and one side is different, with $5 \times 4 = 20$ possibilities.

(iii) All three sides are different, with $\dfrac{5 \times 4 \times 3}{6} = 10$ possibilities.

So there are $5 + 20 + 10 = 35$ possibilities in all,

hence (35).

Note: The calculation in (iii) relies on the observation that if we are choosing the sides in order, there are 5 possibilities for the first side, then 4 for the second, then 3 for the third. However, in these $5 \times 4 \times 3 = 60$ possibilities, each selection xyz will appear 6 times: $xyz, xzy, yxz, yzx, zxy, zyx$. This idea appears in the general formula for $\binom{n}{m}$, the number of ways of choosing m objects from n objects.

30. If m is a cousin's age last year, it is a factor of 1377 and $m+1$ is a factor of 2016.
When we factorise we get $2016 = 2^5 \times 3^2 \times 7$ and $1377 = 3^4 \times 17$.
Checking possible factors of 1377:

m	1	3	9	17	27	51	81	153	459	1377
$m+1$	2	4	10	18	28	52	82	154	460	1378
Factor of 2016?	✓	✓	×	✓	✓	×	×	×	×	×

So there are 4 possible ages.
To have three of these ages multiplying to 2016, we must include 28 (to get a factor of 7), 18 (to get a factor of 3^2) and then 4.
Checking, $2016 = 28 \times 18 \times 4$ and $1377 = 27 \times 17 \times 3$.
Then two years ago their ages multipled to $26 \times 16 \times 2 = 832$,

hence (832).