Solutions – Senior Division

1. They are $708, 718, 728, 738, 748, 758, 768, 778, 788, 798$,

hence (A).

2. $p^2 - 3q^2 = 7^2 - 3 \times (-4)^2 = 49 - 3 \times 16 = 1$,

hence (E).

3. (Also I8)
The perimeters of P, Q, R and S are $4\sqrt{2}$, 8, $2\sqrt{2}$ and 4 respectively,

hence (B).

4. Solving, $7n \geq 194$, and so $n \geq \frac{194}{7} = 27\frac{5}{7}$. So the smallest n can be is 28, and any value $n \geq 28$ is also a solution,

hence (C).

5. The table top is a circle with radius 3 metres, so its area in square metres is $\pi(3)^2 = 9\pi$. Because π is slightly greater than 3, 9π is slightly greater than 27. Of the given choices, 30 is closest,

hence (B).

6. (Also I11)
Alternative 1
The sum of the exterior angles of the pentagon is $360 = 90 + 4 \times (180 - x)$, so that $180 - x = 270/4 = 67.5$ and $x = 180 - 67.5 = 112.5$,

hence (E).

Alternative 2
The sum of the interior angles of the pentagon is $3 \times 180 = 90 + 4x$, so that $x = 450 \div 4 = 112.5$,

hence (E).

7. (Also I12)
If C is the point directly below A and to the left of B, then the right triangle ABC has sides 16 and 12, and hypotenuse x. Then $x^2 = 16^2 + 12^2 = 400$ and $x = 20$,

hence (A).

8. Square both sides of the equation to obtain:

$$\sqrt{x^2+1} = x + 2 \quad \Longrightarrow \quad x^2 + 1 = x^2 + 4x + 4 \quad \Longrightarrow \quad 4x = -3.$$

Hence, the only possible solution is $x = -\frac{3}{4}$. We substitute this value into the original equation and verify that both sides are indeed equal to $\frac{5}{4}$,

hence (B).

9. The probability that the first croissant is not chocolate is $\frac{2}{3}$. When the first croissant is not chocolate, the probability that the second croissant is not chocolate is slightly less than $\frac{2}{3}$. Consequently the probability that both croissants are not chocolate is slightly less than $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$,

hence (B).

Note: We can check that being *slightly less than* does not make the probability closer to the next smaller option, which is $\frac{1}{3}$. With 2 croissants of each type, the second probability is $\frac{3}{5}$, and the overall probability is $\frac{2}{5}$, which is closer to $\frac{4}{9}$ than it is to $\frac{1}{3}$. With more than 2 croissants of each type, the second probability will be even closer to $\frac{2}{3}$ and so the overall probability will be closest to $\frac{4}{9}$.

10. *Alternative 1*

When $n = 1$, the expressions evaluate to $A = 1$, $B = 3$, $C = 4$, $D = 4$ and $E = 3$. Only $B = n^3 + 2n$ and $E = n^2 + 2$ are possible. When $n = 2$, $B = 12$ and $E = 6$, and when $n = 3$, $B = 33$ and $E = 11$. So only B remains possible.

To see that B is always a multiple of 3, write $B = n(n^2-1)+3n = n(n-1)(n+1)+3n$. No matter what n is, one of n, $n-1$ or $n+1$ will be a multiple of 3, so that B is a multiple of 3,

hence (B).

Alternative 2

Options A, C, D and E can be eliminated as above.

To see that $B_n = n^3 + 2n$ is always a multiple of 3, note that

$$B_{n+1} - B_n = (n+1)^3 + 2(n+1) - n^3 - 2n = 3n^2 + 3n + 3$$

so once we know that $B_1 = 3$ is a multiple of 3, then so are all subsequent values,

hence (B).

11. $2^{2016} - 2^{2015} = 2 \times 2^{2015} - 2^{2015} = 2^{2015}$,

hence (C).

12. Suppose n trains stop at neither Oaklands (O) nor Brighton (B).

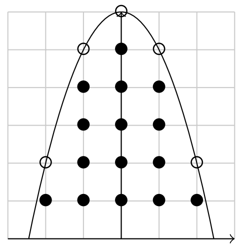
Then $45 - n$ stop at Oaklands but not Brighton. Then $60 - (45 - n) = 15 + n$ stop at Brighton but not Oaklands.

So $60 + 45 - n = 105 - n$ stop at Oaklands and $15 + n + n = 15 + 2n$ do not stop at Oaklands. The number of trains is $120 + n$ of which $24 + \frac{n}{5}$ stop at Oaklands. Hence $15 + 2n = 24 + \frac{n}{5}$, so $75 + 10n = 120 + n$ and $9n = 45$. Then $n = 5$,

	O	Not O
B	60	$15+n$
Not B	$45-n$	n
	$105-n$	$15+2n$

hence (D).

13. The parabola passes through grid points $(0, 6)$, $(\pm 1, 5)$ and $(\pm 2, 2)$ as shown. The number of grid points can be counted in columns, giving a total of $1+4+5+4+1 = 15$ grid points inside the shaded area,



hence (D).

14. In $\triangle SPQ'$, $\angle PSQ' = 90°$ and $PQ' = 2PS$, so that $\triangle SPQ'$ is a 30–60–90 degree triangle. Then $\angle SPQ = 60°$, $\angle QPQ' = 30°$, $\angle QPX = 15°$ and $\angle SPX = 75°$,

hence (E).

15. (Also I19)
Suppose that m is the average number of correct answers by the seven students whose marks weren't listed. Then we know that m is an integer and $10 \le m \le 20$. The average number of correct answers by all ten students is

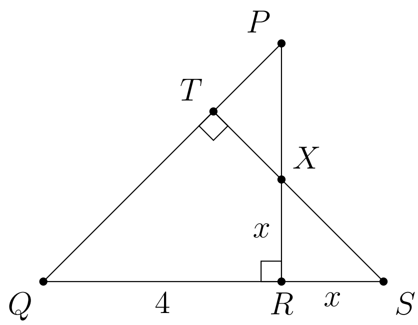
$$\frac{8+8+9+7m}{10} = \frac{25+7m}{10}$$

So $25 + 7m$ is divisible by 10, which is possible only if m is an odd multiple of 5. However, $10 \le m \le 20$, so that $m = 15$.
Therefore, the average number of correct answers by all ten students is

$$\frac{25 + 7 \times 15}{10} = 13$$

hence (D).

16. With all measurements in centimetres, we have $QR = PR = QT = TS = 4$, so that with $x = RS = RX$ as in the diagram, the area of $QRXT$ is $\frac{1}{2} \times 4^2 - \frac{1}{2}x^2 = 8 - \frac{1}{2}x^2$.
By Pythagoras' theorem, $PQ^2 = 4^2 + 4^2 = 32$ and $PQ = QS = 4\sqrt{2}$.
Then $x = 4\sqrt{2} - 4 = 4(\sqrt{2} - 1)$ so the area of $QRXT$ is



$$8 - \frac{1}{2}x^2 = 8 - 8(\sqrt{2} - 1)^2 = 16(\sqrt{2} - 1)$$

hence (A).

17. Adding the arithmetic progression,

$$S = x + 2x + \cdots + 100x = x(1 + 2 + \cdots + 100) = 5050x = 2 \times 5^2 \times 101x.$$

If S is a square, only even powers can occur in the prime factorisation, so that the smallest x can be is $2 \times 101 = 202$,

hence (A).

18. *Alternative 1*

If the radius of the original spheres is r, then the radius of the new ones is $0.8r$. The ratio of smaller to larger volumes is therefore

$$\frac{\frac{4}{3}\pi(0.8r)^3}{\frac{4}{3}\pi r^3} = (0.8)^3 = 0.512$$

If the ratio were exactly 0.5, there would be exactly 20 smaller spheres. Since the ratio is slightly more than 0.5, there will be 19 smaller spheres with some leftover gold.

Checking, $19 \times 0.512 = 9.728$, so that 19 of the smaller spheres, can be cast from the original 10 spheres,

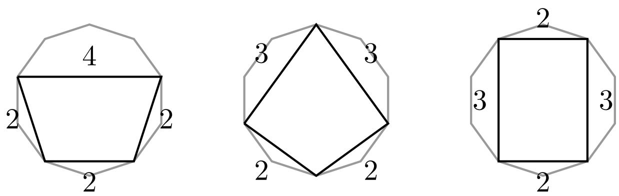
hence (E).

Alternative 2

Due to the way volume changes when all linear dimensions are scaled, the volume of a smaller sphere is $\left(\frac{4}{5}\right)^3 = \frac{64}{125}$ that of an original sphere. That is, each larger sphere is $\frac{125}{64}$ smaller spheres, and 10 larger spheres is $\frac{1250}{64} = \frac{625}{32} = 19\frac{17}{32}$ smaller spheres. So 19 smaller spheres can be made,

hence (E).

19. Each edge of the quadrilateral spans across 2 or more edges of the decagon. Going around the quadrilateral, label each edge with this number of decagon edges, giving 4 numbers (h, i, j, k), each 2 or more, with $h+i+j+k = 10$. By choosing a starting edge and direction, we can assume that h is the largest and that $i \geq k$. The possibilities are $(4, 2, 2, 2)$, $(3, 3, 2, 2)$ and $(3, 2, 3, 2)$, corresponding to these quadrilaterals:



hence (B).

20. The sequence begins $1, 2, 4, 7, 10, 13, \ldots$. Ignoring 2, terms of the sequence are precisely those which are one more than a multiple of 3, as argued below. There are $2016 \div 3 = 672$ such numbers less than 2016 (taking care to include 1 instead of 2017), hence there are 673 such numbers in total, including the 2.

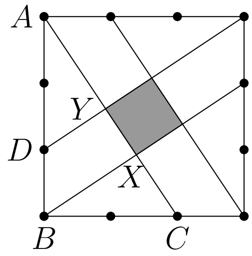
We now prove that the sequence consists of 2, together with all numbers of the form $3k+1$. Suppose that for $n \geq 2$, the first $n+1$ terms of the sequence consist of 2 and all numbers of the form $3k+1$ where $0 \leq k \leq n-1$. This is true for $n = 2$.

Write $m = 3n - 2$ so that $m \geq 4$. Since $1, 2$ and m are in the sequence, the next term is neither $m+1$ or $m+2$.

The number $m+3 = 3n+1$ is one more than a multiple of 3, and is greater than 4. However the possible sums of pairs of numbers are $2+2 = 4$, $2+(3k+1) = 3(k+1)$ and $(3k+1)+(3j+1) = 3(j+k)+2$. Of these, only 4 is one more than a multiple of 3, but it is less than $m+3$. So $m+3$ is not the sum of two terms in the sequence, and $m+3 = 3n+1$ must be the next term. By induction, term $(n+2)$ in the sequence is $3n+1$, for all $n \geq 2$,

hence (E).

21. *Alternative 1*



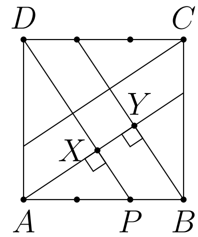
Let the outer square have side $3x$. Then $AC = \sqrt{9x^2 + 4x^2} = x\sqrt{13}$. Triangles $\triangle ABC$, $\triangle AXB$ and $\triangle AYD$ are similar, so $\dfrac{AX}{AB} = \dfrac{AB}{AC}$ and $AX = \dfrac{9x^2}{x\sqrt{13}} = \dfrac{9}{\sqrt{13}}x$.

Similarly $AY = \dfrac{6}{\sqrt{13}}x$ so that $XY = \dfrac{3}{\sqrt{13}}x$.

But $XY = 1$ so that $x = \dfrac{\sqrt{13}}{3}$. Then the side of the large square is $3x = \sqrt{13}$ and the area of the large square is 13 square units,

hence (D).

Alternative 2



Triangles $\triangle AXP$ and $\triangle AYB$ are similar with $AX = YB$ and $AP = \frac{2}{3}AB$. Hence $AX = \frac{2}{3}AY$ and $XY = \frac{1}{3}AY$.

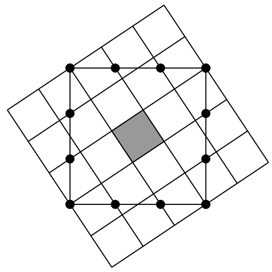
But $XY = 1$ so that $AY = 3$ and $AX = 2$.

So $\triangle AYB$ has $AY = 3$ and $YB = 2$, so its area is $\dfrac{3 \times 2}{2} = 3$.

Square $ABCD$ consists of the central square and four triangles congruent to $\triangle AYB$, so its total area is $1 + 4 \times 3 = 13$ square units,

hence (D).

Alternative 3



Continue to draw parallels at equal spacing to the sides of the small square.

Then the grid passes through all the trisection points as shown, so the vertices of the large square lie on grid lines. By counting squares or subdividing areas, the area of the original large square is 13 square units,

hence (D).

22. *Alternative 1*

Let $n^2 + n + 34 = (n + k)^2 = n^2 + 2kn + k^2$ where $k > 0$ is an integer. Then $n = \dfrac{34 - k^2}{2k - 1}$, so if $k > 5$, then $n < 0$. The only values of k that result in integer values of n are 1, 2, 3, and 5. The corresponding values of n are 33, 10, 5, and 1, and the required sum is $33 + 10 + 5 + 1 = 49$,

hence (E).

Alternative 2

The expression $n^2 + n + 34$ is always even, and so suppose $(2a)^2 = n^2 + n + 34$ where $a > 0$. Completing the square,

$$4a^2 = \left(n + \frac{1}{2}\right)^2 - \frac{1}{4} + 34$$

$$16a^2 = (2n + 1)^2 + 135$$

$$(4a)^2 - (2n + 1)^2 = 135$$

$$(4a + 2n + 1)(4a - 2n - 1) = 3^3 \times 5$$

So 135 factorises into a product of two integers $x = 4a + 2n + 1$ and $y = 4a - 2n - 1$. Clearly $x > 0$ and $x > y$, and $y = \dfrac{135}{x} > 0$.

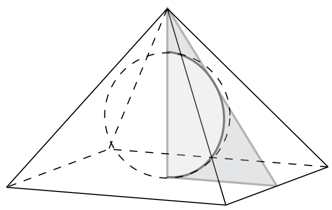
From the prime factorisation of $135 = 3^3 \times 5$, there are four solutions to $xy = 135$ where $x > y > 0$. From these values for $a = \frac{x+y}{8}$ and $n = \frac{x-y-2}{4}$ can be found.

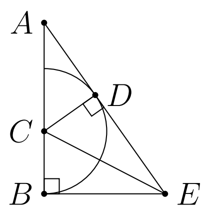
$x = 4a + 2n + 1$	$y = 4a - 2n - 1$	a	n
135	1	17	33
45	3	6	10
27	5	4	5
15	9	3	1
			49

From the table, the sum of the 4 possible values of n is 49,

hence (E).

23. Consider a cross-section made by a triangle through A, the apex, B, the centre of the base and E, the midpoint of one side of the square base. This includes C, the centre of the sphere and D, the point at which the sphere touches AE.





Then $BE = 1$, $AE = \sqrt{3}$ and $AB = \sqrt{2}$. Since EB and ED are both tangents, $EB = ED = 1$ so that $AD = \sqrt{3} - 1$. Since triangles $\triangle ABE$ and $\triangle ADC$ are similar, the radius of the sphere is

$$CD = AD \cdot \frac{BE}{AB} = \frac{\sqrt{3}-1}{\sqrt{2}} = \frac{\sqrt{3}-1}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}-\sqrt{2}}{2}$$

hence (C).

24. If all 10 cards were green, then each card would have at least one adjacent card with a smaller number. This is not possible for the smallest number of the ten.

However, it is possible to have 9 green cards, and this can be done in many ways.

For instance, number the cards from $n = -4$ to $n = 5$, and write $100 - n^2$ on each card:

84	91	96	99	100	99	96	91	84	75

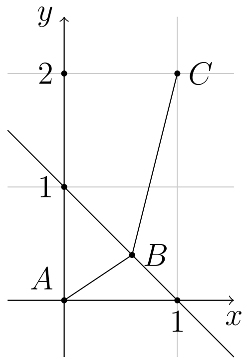
For $-4 \le n \le 4$, the parabola $y = f(n) = 100 - n^2$ is concave down, giving the required inequality. We can check that

$$\frac{f(n-1)+f(n+1)}{2} = \frac{100-(n+1)^2+100-(n-1)^2}{2} = 99 - n^2 = f(n) - 1$$

to confirm that 9 green cards is the largest possible number of green cards,

hence (E).

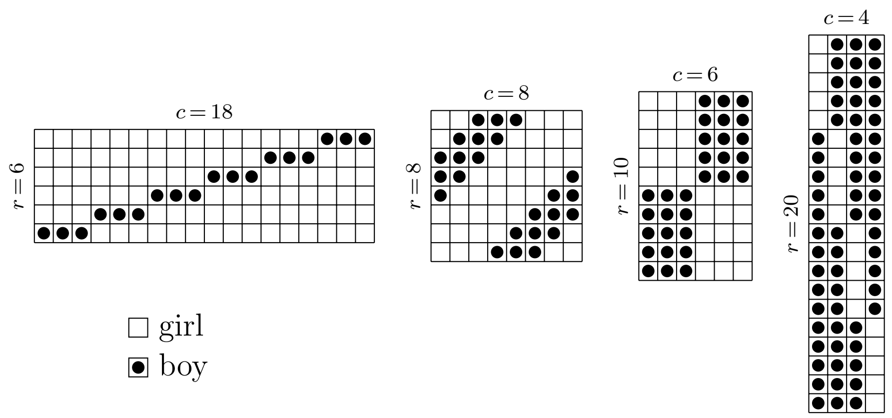
25. The value $\sqrt{x^2+(1-x)^2}$ is the distance in the coordinate plane between points $A=(0,0)$ and $B=(x,1-x)$. The value $\sqrt{(1-x)^2+(1+x)^2}$ is the distance between points $B=(x,1-x)$ and $C=(1,2)$.



So the task is to minimise $AB+BC$ where B lies on the line $y=1-x$. The minimum is clearly $AC=\sqrt{5}$, which occurs when ABC is a straight line,

hence (C).

26. (Also I29)
Let the formation have r rows and c columns, so the size of the band is rc.
The band contains $3r$ boys and $5c$ girls, so the size of the band is also $3r+5c$.
Then $rc-3r-5c=0$, or equivalently, $(r-5)(c-3)=15$.
The positive integer solutions for the ordered pair $(r-5,c-3)$ are $(1,15)$, $(3,5)$, $(5,3)$, and $(15,1)$. There are negative integer factorisations of 15, but these have $r\le 0$ or $c\le 0$.
The corresponding solutions for (r,c) are $(6,18)$, $(8,8)$, $(10,6)$, and $(20,4)$, and the corresponding values of rc are 108, 64, 60, and 80.
For each of these sizes, there is at least one arrangement of boys and girls:



Therefore the sum of all possible band sizes is $108+64+60+80=312$,

hence (312).

27.

$$f(n)-f(m)=(an^2+bn+c)-(am^2+bm+c)$$
$$2016^2-0=(a(n+m)+b)(n-m)$$

Then $n-m$ is a positive divisor of 2016^2.
Moreover, for each positive divisor d of 2016^2, the values $m=0$, $n=d$, $a=0$, $b=2016^2/d$ show that d is a possible value of $n-m$.

The number of positive divisors of $2016^2 = 2^{10}\,3^4\,7^2$ is $11 \times 5 \times 3 = 165$, and so there are 165 possible values of $n - m$,

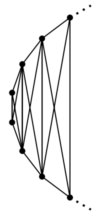
hence (165).

28. Rewriting the equation in index form, we have $a^{\sqrt{b}} = a^{b/2}$. If $a = 1$, then b can take any value from 1 to 100. If $a \neq 1$, then b must satisfy $\sqrt{b} = b/2$ and $b > 0$, so $b = 4$. Then a can take any value from 2 to 100. Hence there are 199 solutions in total,

hence (199).

29. (Also I30)

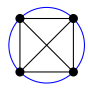
Alternative 1



Draw the 64-gon and all 30 diagonals parallel to a fixed side, dividing it into 31 trapeziums. In each trapezium, draw both diagonals. This requires $64 + 30 + 2 \times 31 = 156$ chords. So the maximum number of chords is 156 or more.

In fact, the maximum is 156, as is shown below.

Firstly, for n points on a circle, where n is even, the same argument tells us that if M_n is the maximum number of chords, then $M_n \geq n + \frac{n}{2} - 2 + 2 \times (\frac{n}{2} - 1) = \frac{5}{2}n - 4$.



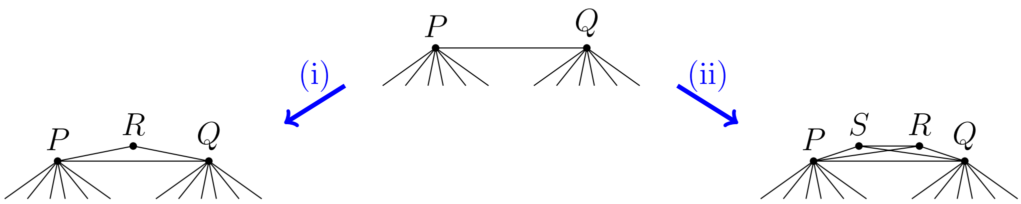
Claim: For all even values of n, $M_n = \frac{5}{2}n - 4$.

Clearly when $n = 4$, $M_n = 6 = \frac{5}{2} \times 4 - 4$, so the claim is true.

For a diagram with a chord PQ on the boundary, consider the following two possible steps:

(i) Between P and Q, add a point R and two chords PR and QR.

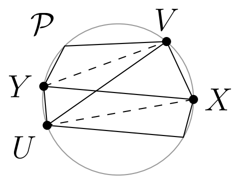
(ii) Between P and Q, add two points R, S and five chords PR, PS, QR, QS, RS.



Step (ii) gives more chords per point, so a construction that starts with the 4-point diagram above and builds up using steps (i) and (ii) will have the greatest number of chords if it uses step (ii) as much as possible. For an even number of points, it will only use step (ii). Such a diagram will have $6 + 5 \times \frac{1}{2}(n - 4) = \frac{5}{2}n - 4$ chords.

The only uncertainty is whether a diagram with the maximum number of chords can be built from the $n = 4$ diagram using only steps (i) and (ii).

Suppose the maximum number of chords are drawn. Any chord in the diagram is either an edge of the outer n-gon, a crossed chord or an uncrossed chord. The uncrossed chords divide the n-gon into smaller polygons. If these were triangles and quadrilaterals then the n-gon could be built up using steps (i) and (ii), adding one polygon at a time.



Consider such a polygon $\mathcal{P}$ with each side an uncrossed chord and with 5 or more sides. Some diagonal XY must be a chord in the diagram, since the diagram has the maximum number of chords. Then XY must be crossed by another chord UV, or else the uncrossed chord XY would have split $\mathcal{P}$ into smaller polygons. Any chord passing through any edge of quadrilateral $XUYV$ would cross XY or UV, which would then be double-crossed, which is forbidden. So every edge of $XUYV$ crosses no chords, so it must be in the diagram (due to maximality) where it will be an uncrossed chord.

Now, since $\mathcal{P}$ has 5 or more sides, $XUYV \neq \mathcal{P}$ so at least one side of $XUYV$, say XU, is not a side of $\mathcal{P}$. Then XU is an uncrossed chord that splits $\mathcal{P}$ into smaller polygons, which cannot happen. Hence such a polygon $\mathcal{P}$ must have 4 or fewer edges.

In conclusion, any diagram with the maximum number of chords can be built from the 4-point diagram using steps (i) and (ii). When n is even, the most chords are obtained using only step (ii), which gives $M_n = \frac{5}{2}n - 4$. Consequently $M_{64} = 160 - 4 = 156$,

hence (156).

Alternative 2

As in the first solution, 156 chords are possible.

To see that this is the most, we first note a well-known result, known as *triangulation of a polygon*:

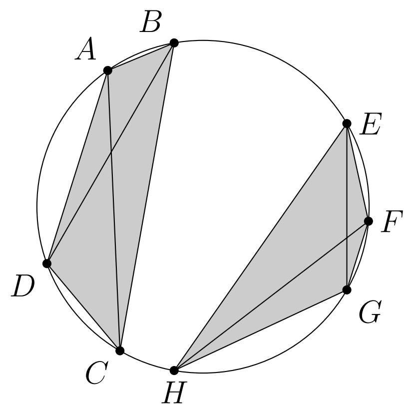
> When a polygon with n sides (n-gon) is cut into triangles, where each triangle's vertices are vertices of the original n-gon, there are $n-2$ triangles. Also, there are $n-3$ cuts, each along a diagonal of the n-gon.

A first consequence of triangulation is that the maximum number of non-intersecting diagonals that can be drawn inside an n-gon is $n - 3$.

Secondly, for triangles whose vertices are vertices of the original n-gon, the maximum number of non-overlapping triangles that can be drawn inside the n-gon is $n-2$. This is because we can add more triangles to get a triangulation.

Thirdly, for quadrilaterals whose vertices are vertices of the original n-gon, the maximum number of non-overlapping quadrilaterals that can be drawn inside the n-gon is $\frac{n-2}{2} = \frac{n}{2} - 1$. This is because each quadrilateral can be split into two non-overlapping triangles.

Returning to the question, for every pair of crossing chords AC and BD, shade in the quadrilateral $ABCD$. For two shaded quadrilaterals $ABCD$ and $EFGH$, neither diagonal AC or BD intersects EG or FH, so $ABCD$ and $EFGH$ do not overlap.



That is, the shaded quadrilaterals are non-overlapping, and so there are at most $\frac{n}{2} - 1 = 31$ of them. Thus there are at most 31 pairs of crossing chords.

For each pair of crossing chords, remove one chord. There are at most 31 removed chords. The chords remaining have no crossings, and are either sides of the 64-gon (at most 64 of these) or diagonals of the 64-gon (at most 61 of these). Consequently the number of chords originally was at most $64 + 61 + 31 = 156$,

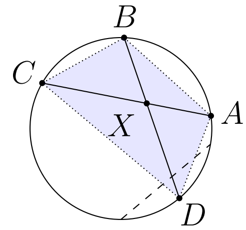
hence (156).

Alternative 3

As in the first solution, 156 chords can be drawn.

To see that 156 is the maximum, suppose the maximum number of chords are drawn—no more can be added. In particular, any possible chord that intersects no other drawn chord must be drawn. This includes all edges of the regular 64-gon. It also means that the only polygons that have all vertices on the circle and no chords inside are triangles, since otherwise a diagonal could be drawn.

When two of the chords AC and BD intersect at an interior point X, there are no other chords intersecting AC and BD, so no other chord will pass inside the quadrilateral $ABCD$. Consequently, the chords AB, BC, CD and DA do not intersect any other chords, so they must be included. Then $ABCD$ appears in the diagram as a *crossed quadrilateral*: all sides and both diagonals are drawn.



So that we can use Euler's formula $f + v = e + 2$, we consider the figure as a planar graph where the vertices include the 64 original points and the intersection points, and the edges include the chords that aren't cut by another chord and the two parts of the chords that are cut by another chord.

There are three types of faces: the exterior of the 64-gon, triangles that are part of a crossed quadrilateral, and triangles that have all vertices on the circle. Suppose there are q crossed quadrilaterals and t triangles. Then

- The number of vertices is $v = 64 + q$, the 64 initial vertices plus one for each crossed quadrilateral.
- The number of faces is $f = 1 + t + 4q$, the outside of the 64-gon, the t triangles, and 4 triangles for each crossed quadrilateral.
- The number of vertices is e where $2e = 64 + 3\,(t + 4q)$. This total is from adding the number of edges on each face, which counts each edge twice.
- The number of chords is $c = e - 2q = 32 + \frac{3}{2}t + 4q$, since for each crossed quadrilateral the number of edges is 2 more than the number of chords.

Then in Euler's formula $f + v = e + 2$:

$$\begin{aligned} 0 = (f + v) - (e + 2) &= (65 + t + 5q) - (34 + \frac{3}{2}t + 6q) \\ &= 31 - \frac{1}{2}t - q \\ c &= 32 + \frac{3}{2}t + 4(31 - \frac{1}{2}t) \\ &= 156 - \frac{1}{2}t \end{aligned}$$

Hence the maximum number of chords drawn is 156, attained when $q = 31$ and $t = 0$, hence (156).

30. *Alternative 1*

Observe that

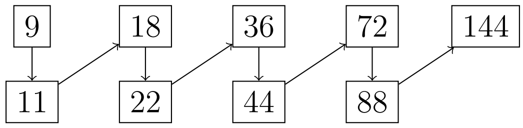
$$\begin{aligned} f(4n+1) &= 4n+3 \\ f(4n+3) &= f(f(4n+1)) = 2(4n+1) \\ f(2(4n+1)) &= f(f(4n+3)) = 2(4n+3) \\ f(2(4n+3)) &= f(f(2(4n+1))) = 2^2(4n+1) \\ f(2^2(4n+1)) &= f(f(2(4n+3))) = 2^2(4n+3) \\ &\vdots \quad \vdots \\ f(2^k(4n+1)) &= 2^k(4n+3) \\ f(2^k(4n+3)) &= 2^{k+1}(4n+1) \end{aligned}$$

So that $f(2016) = f(63 \times 32) = f((4 \times 15 + 3) \times 2^5) = 61 \times 2^6 = 3904$,

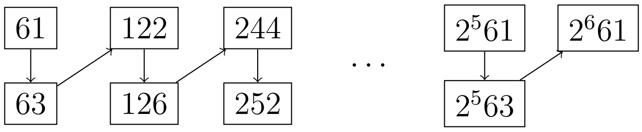
hence (904).

Alternative 2

Place the positive integers in the plane and draw arrows between numbers m and $f(m)$. So there are arrows $4n+1 \to 4n+3$ that link pairs of odd numbers, and also whenever $m \to x$, also $x \to 2m$. For instance, with $n = 2$ we get $4n+1 = 9$, $4n+3 = 11$ and then



Since $2016 = 63 \times 2^5$ where $63 = 2 \times 15 + 3$, we have



Then $f(2016) = 2^6 \times 61 = 3904$,

hence (904).

Alternative 3

We have $f(f(f(n))) = f(2n)$ but also $f(f(f(n))) = 2f(n)$, so that $f(2n) = 2f(n)$ and by iterating k times, $f(2^k n) = 2^k f(n)$. Hence $f(2016) = f(2^5\,63) = 2^5 f(63)$. Also $f(63) = f(f(61)) = 122$ so that $f(2016) = 2^5 \times 122 = 3904$,

hence (904).