# Singapore Mathematical Society

## Singapore Mathematical Olympiad (SMO) 2013
### (Junior Section, Round 2)

1. Let $a < b < c < d < e$ be real numbers. Among the 10 sums of the pairs of these numbers, the least three are 32, 36 and 37 while the largest two are 48 and 51. Find all possible values of $e$.

2. In the triangle $ABC$, points $D, E, F$ are on the sides $BC, CA$ and $AB$ respectively such that $FE$ is parallel to $BC$ and $DF$ is parallel to $CA$. Let $P$ be the intersection of $BE$ and $DF$, and $Q$ the intersection of $FE$ and $AD$. Prove that $PQ$ is parallel to $AB$.

3. Find all primes that can be written both as a sum of two primes and as a difference of two primes.

4. Let $a$ and $b$ be positive integers with $a > b > 2$. Prove that $\frac{2^a+1}{2^b-1}$ is not an integer.

5. Six musicians gathered at a chamber music festival. At each scheduled concert some of the musicians played while the others listened as members of the audience. What is the least number of such concerts which would need to be scheduled so that for every two musicians each must play for the other in some concert?