AUSTRALIAN MATHEMATICS COMPETITION

Intermediate Years 9 & 10

(Australian school years)

THURSDAY 1 AUGUST 2019

AUSTRALIAN MATHS TRUST

NAME: ______________________________

TIME ALLOWED: 75 minutes

INSTRUCTIONS AND INFORMATION

General

1 Do not open the booklet until told to do so by your teacher.

2 NO calculators, maths stencils, mobile phones or other calculating aids are permitted. Scribbling paper, graph paper, ruler and compasses are permitted, but are not essential.

3 Diagrams are NOT drawn to scale. They are intended only as aids.

4 There are 25 multiple-choice questions, each requiring a single answer, and 5 questions that require a whole number answer between 0 and 999. The questions generally get harder as you work through the paper. There is no penalty for an incorrect response.

5 This is a competition not a test; do not expect to answer all questions. You are only competing against your own year in your own country/Australian state so different years doing the same paper are not compared.

6 Read the instructions on the answer sheet carefully. Ensure your name, school name and school year are entered. It is your responsibility to correctly code your answer sheet.

7 When your teacher gives the signal, begin working on the problems.

The answer sheet

1 Use only lead pencil.

2 Record your answers on the reverse of the answer sheet (not on the question paper) by FULLY colouring the circle matching your answer.

3 Your answer sheet will be scanned. The optical scanner will attempt to read all markings even if they are in the wrong places, so please be careful not to doodle or write anything extra on the answer sheet. If you want to change an answer or remove any marks, use a plastic eraser and be sure to remove all marks and smudges.

Integrity of the competition

The AMT reserves the right to re-examine students before deciding whether to grant official status to their score.

Reminder: You may sit this competition once, in one division only, or risk no score.

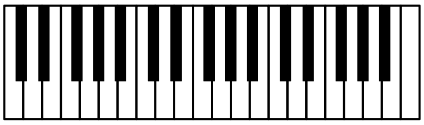
Intermediate Division

Questions 1 to 10, 3 marks each

1. The value of $20.19 - 19$ is

(A) 39.19 (B) 20.38 (C) 20 (D) 1.19 (E) 1

2. Sharyn's piano lesson was 40 minutes long, and finished at 4.10 pm. When did it start?

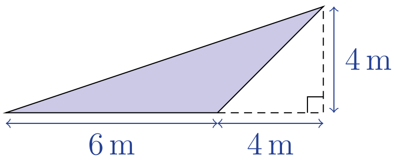


(A) 3.30 pm (B) 3.40 pm (C) 3.50 pm (D) 4.40 pm (E) 4.50 pm

3. Which of the following is closest to 7×1.8?

(A) 10 (B) 11 (C) 12 (D) 13 (E) 14

4. What is the area of the shaded triangle?



(A) $8\,\text{m}^2$ (B) $12\,\text{m}^2$ (C) $14\,\text{m}^2$ (D) $20\,\text{m}^2$ (E) $24\,\text{m}^2$

5. Five-eighths of a number is 200. What is the number?

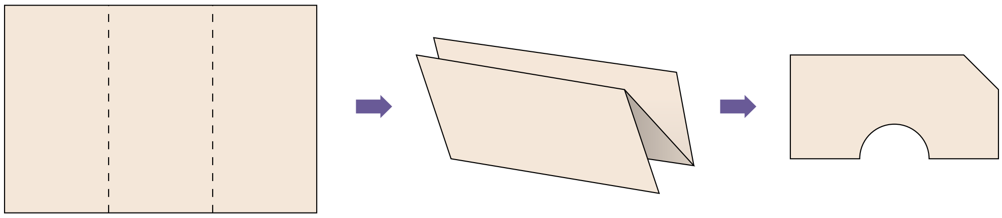
(A) 120 (B) 320 (C) 275 (D) 75 (E) 280

6. Every row and every column of this 3×3 square must contain each of the numbers 1, 2 and 3.
What is the value of $N + M$?

1		
	2	N
		M

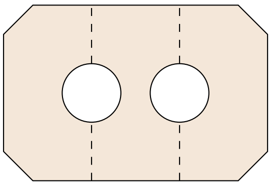
(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

7. A piece of paper is folded in three, then a semi-circular cut and a straight cut are made, as shown in the diagram.

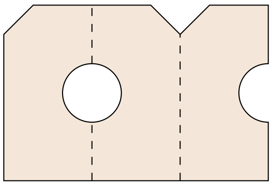


When the paper is unfolded, what does it look like?

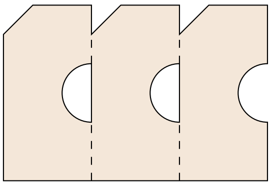
(A)



(B)



(C)



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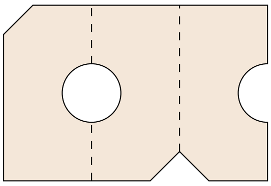
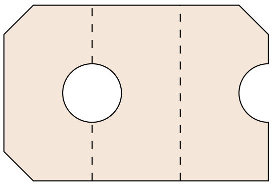


(E)



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8. When a rectangle is cut in half, two squares are formed. If each square has a perimeter of 48, what is the perimeter of the original rectangle?

(A) 96 (B) 72 (C) 36 (D) 24 (E) 12

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9. Consider the *undulating* number sequence

$$1, 4, 7, 4, 1, 4, 7, 4, 1, 4, \ldots ,$$

which repeats every four terms. The running total of the first 3 terms is 12. The running total of the first 7 terms is 28.

Which one of the following is also a running total of this sequence?

(A) 61 (B) 62 (C) 67 (D) 66 (E) 65

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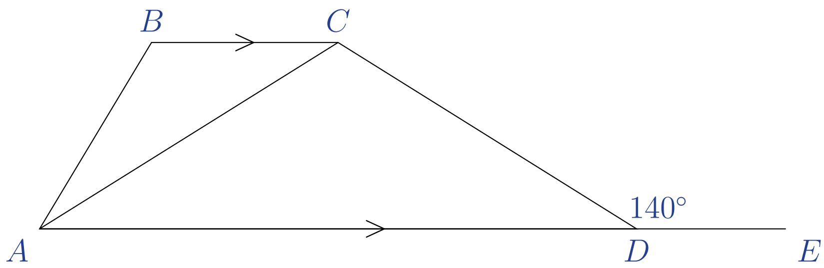
10. Sebastien has a Personal Identification Number (PIN) consisting of four digits. The first three digits in order are 591. If Sebastien's PIN is divisible by 3, then how many possibilities are there for the final digit?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

Questions 11 to 20, 4 marks each

11. A quadrilateral $ABCD$ has $AD \| BC$, $AB = BC$ and $AC = CD$. The external $\angle CDE = 140°$. What is the value, in degrees, of $\angle ABC$?



(A) 90 (B) 100 (C) 110 (D) 120 (E) 130

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12. In my dance class, 14 students are taller than Bob, and 12 are shorter than Alice. Four students are both shorter than Alice and taller than Bob. How many students are in my dance class?

(A) 22 (B) 24 (C) 26 (D) 28 (E) 30

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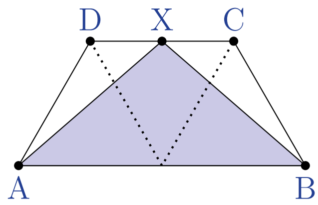
13. Three equilateral triangles are joined to form the quadrilateral $ABCD$ shown.

The point X is halfway along CD.

What fraction of the area of $ABCD$ is shaded?



(A) $\frac{1}{2}$ (B) $\frac{3}{4}$ (C) $\frac{2}{3}$ (D) $\frac{5}{6}$ (E) $\frac{3}{5}$

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14. In a year 10 Maths class, there are 30 students. Each student is either 15 or 16 years old, and either left- or right-handed.

The ratio of right-handed students to left-handed students is 4 : 1, the ratio of 15 year olds to 16 year olds is 1 : 2 and the ratio of left-handed 15 year olds to left-handed 16 year olds is 1 : 5.

If the names of the students in this class are placed in a hat and one is selected at random, what is the probability that the student selected is 15 years old and right-handed?

(A) $\frac{1}{30}$ (B) $\frac{1}{6}$ (C) $\frac{3}{10}$ (D) $\frac{1}{2}$ (E) $\frac{4}{5}$

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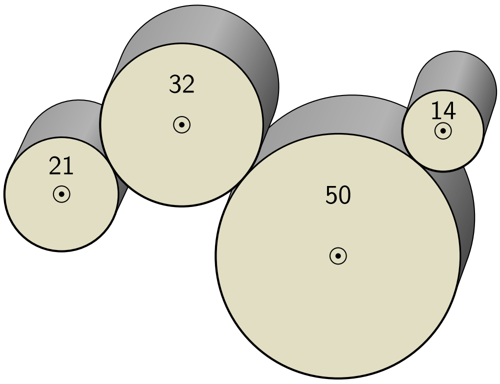
15. The set of four rollers shown has fixed axles and transfers rotation from each roller to the next without slipping. Their diameters are 21 cm, 32 cm, 50 cm and 14 cm respectively.

While the 21 cm roller makes a full rotation (360°), through which angle does the 14 cm roller rotate?



(A) 180° (B) 310° (C) 360° (D) 540° (E) 620°

16. In a box of apples, $\frac{3}{7}$ of the apples are red and the rest are green. Five more green apples are added to the box. Now $\frac{5}{8}$ of the apples are green.

How many apples are there now in the box?

(A) 32 (B) 33 (C) 38 (D) 40 (E) 48

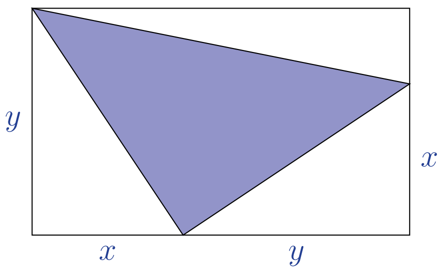
17. Asha chooses a whole number from 1 to 5 and announces it. Then Richy chooses a whole number from 1 to 5 and announces it. Finally, Asha chooses a whole number from 1 to 5 and announces it.

If the sum of the three numbers announced is a multiple of 7, then Asha wins; otherwise, Richy wins.

What number should Asha choose on her first turn to guarantee that she can win?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

18. The area of the shaded triangle inside this rectangle is



(A) $\frac{1}{2}(x+y)^2$ (B) $x(x+y)$ (C) $y(x+y)$

(D) $\frac{1}{2}(y^2-x^2)$ (E) $\frac{1}{2}(x^2+y^2)$

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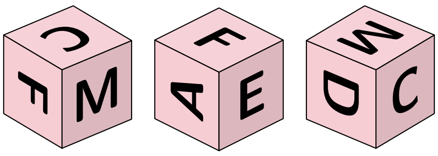
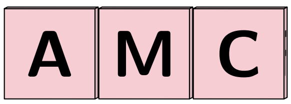
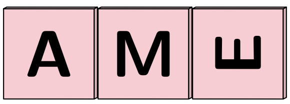
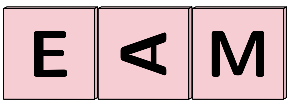
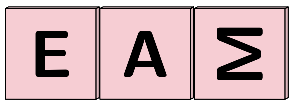
19. These three cubes are labelled in exactly the same way, with the 6 letters A, M, C, D, E and F on their 6 faces:

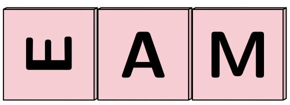
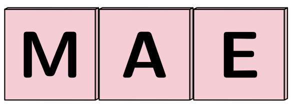


The cubes are now placed in a row so that the front looks like this:



When we look at the cubes from the opposite side, we will see

(A)  (B)  (C) 

(D)  (E) 

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20. Five numbers are placed in a row. From the third number on, each number is the average of the previous two numbers. The first number is 12 and the last number is 7. What is the third number?

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

Questions 21 to 25, 5 marks each

21. A pool can be filled through three pipes that can be used together or separately. If only the first pipe is used, the pool is filled in 21 hours. If only the second pipe is used, the pool is filled in 24 hours. If all three pipes are used, the pool is filled in 8 hours.

How long will it take to fill the pool using only the third pipe?

(A) 12 hours (B) 14 hours (C) 27 hours (D) 28 hours (E) 30 hours

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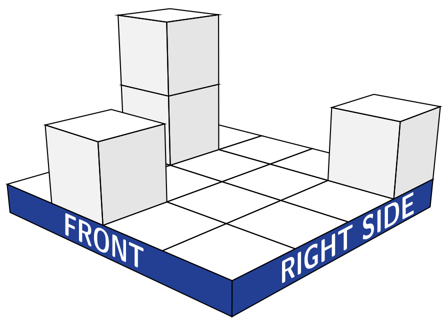
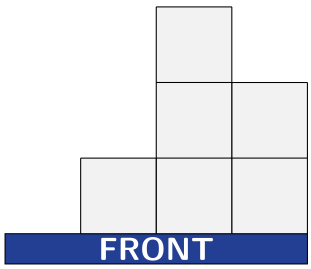
22. A $4\,\text{cm} \times 4\,\text{cm}$ board can have $1\,\text{cm}^3$ cubes placed on it as shown.

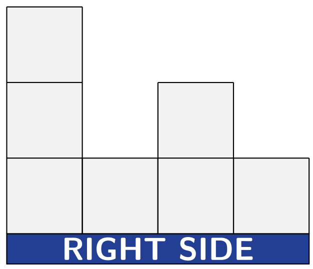


The board is cleared, then a number of these cubes are placed on the grid. The front and right side views are shown.

What is the maximum number of cubes there could be on the board?

(A) 10 (B) 11 (C) 16 (D) 17 (E) 18





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23. Manny has three ways to travel the 8 kilometres from home to work: driving his car takes 12 minutes, riding his bike takes 24 minutes and walking takes 1 hour and 44 minutes. He wants to know how to get to work as quickly as possible in the event that he is riding his bike and gets a flat tyre.

He has three strategies:

(i) If he is close to home, walk back home and then drive his car.
(ii) If he is close to work, just walk the rest of the way.
(iii) For some intermediate distances, spend 20 minutes fixing the tyre and then continue riding his bike.

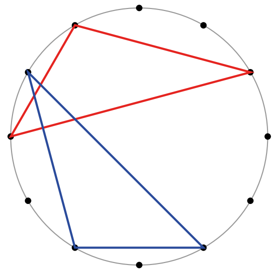
He knows there are two locations along the route to work where the strategy should change. How far apart are they?

(A) 2 km (B) 3 km (C) 4 km (D) 5 km (E) 6 km

24. Out of modern musical theory comes the following question. Twelve points are equally spaced around a circle. Three points are to be joined to make a triangle.

We count two triangles as being the same only if they match perfectly after rotating, but not reflecting. For instance, the two triangles shown are the same.

How many different triangles can be made?



(A) 10 (B) 14 (C) 19 (D) 20 (E) 22

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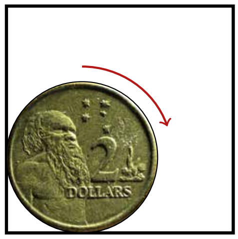
25. A circular coin of radius 1 cm rolls around the inside of a square without slipping, always touching the boundary of the square.

When it returns to where it started, the coin has performed exactly one whole revolution.

In centimetres, what is the side length of the square?



(A) π (B) 3.5 (C) $1+\pi$ (D) 4 (E) $2+\frac{\pi}{2}$

For questions 26 to 30, shade the answer as an integer from 0 to 999 in the space provided on the answer sheet.

Questions 26–30 are worth 6, 7, 8, 9 and 10 marks, respectively.

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26. A positive whole number is called *stable* if at least one of its digits has the same value as its position in the number. For example, 78247 is stable because a 4 appears in the 4^{th} position. How many stable 3-digit numbers are there?

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27. When I divide an integer by 15, the remainder is an integer from 0 to 14. When I divide an integer by 27, the remainder is an integer from 0 to 26.

For instance, if the integer is 100 then the remainders are 10 and 19, which are different.

How many integers from 1 to 1000 leave the same remainders after division by 15 and after division by 27?

28. The number 35 has the property that when its digits are both increased by 2, and then multiplied, the result is $5 \times 7 = 35$, equal to the original number.

Find the sum of all two-digit numbers such that when you increase both digits by 2, and then multiply these numbers, the product is equal to the original number.

29. The Leader of Zip decrees that the digit 0, since it represents nothing, will no longer be used in any counting number. Only counting numbers without 0 digits are allowed.

So the counting numbers in Zip begin 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, ... , where the tenth counting number is 11.

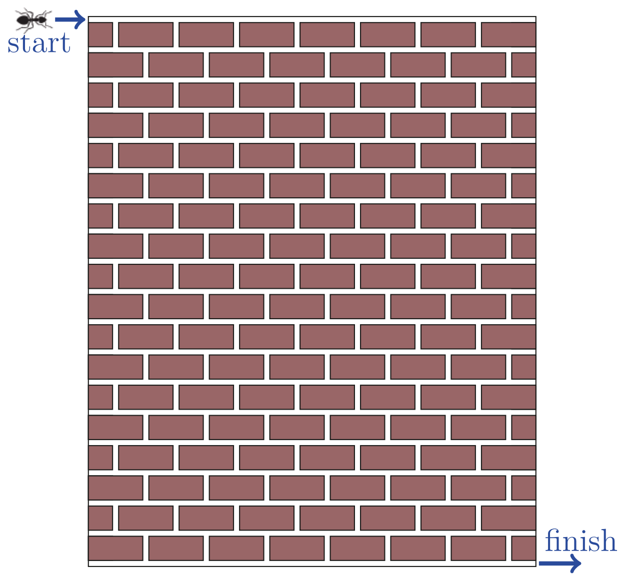
When you write out the first one thousand allowable counting numbers in Zip, what are the last three digits of the final number?

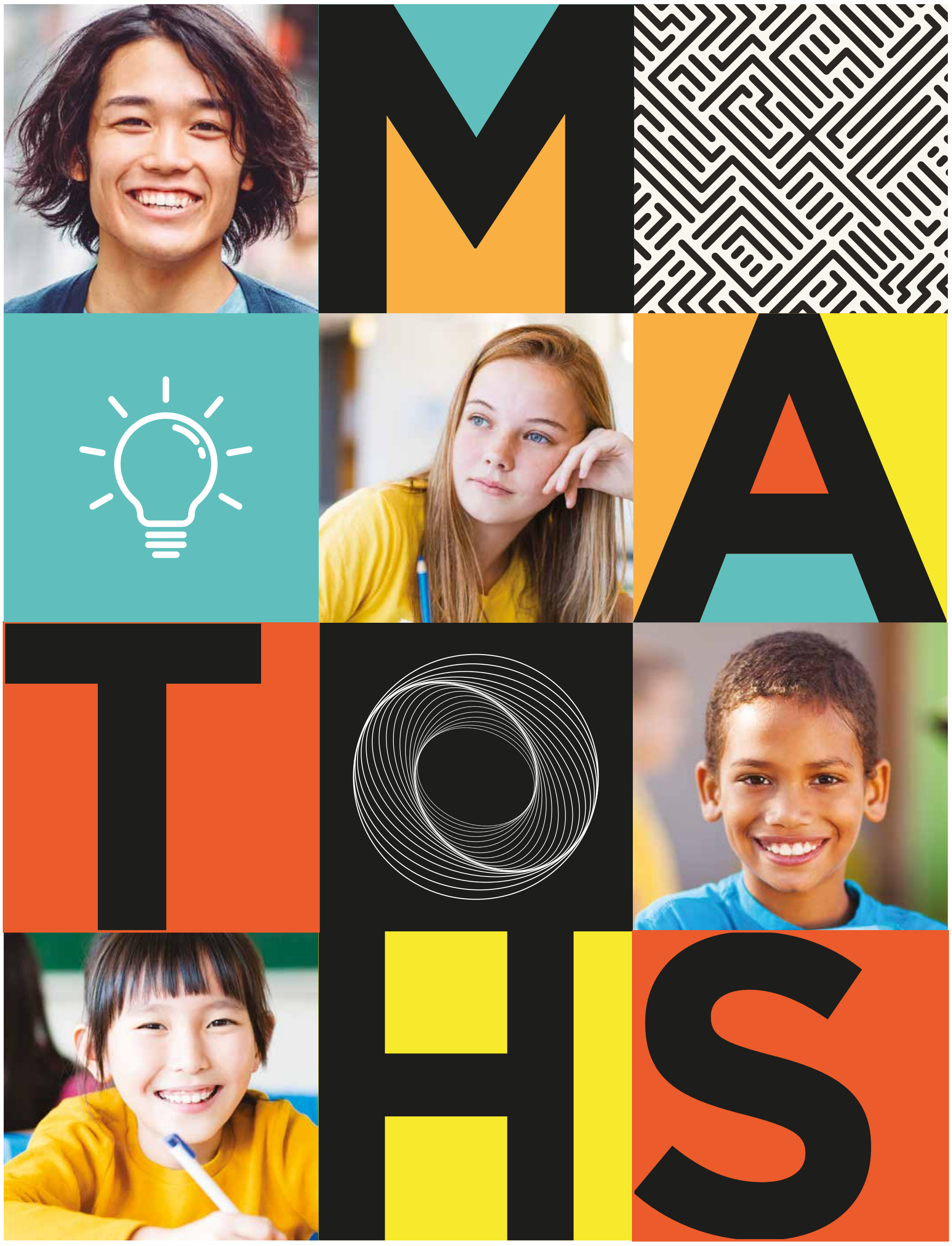
30. Antony the ant is at the top-left corner of this brick wall and needs to get to the bottom-right corner.

Because it is a hot day, he avoids the dark bricks and only walks on the cooler white mortar between the bricks and at the top and bottom of the wall.

There are 18 rows of bricks, each with 7 whole bricks and one half-brick in an alternating pattern.

How many different ways are there for Antony to walk to the opposite corner as quickly as possible?





2019 AMC — INTERMEDIATE

SOLVE PROBLEMS.
CREATE THE FUTURE.

Problems are part of life and we've made it our mission to equip young students with the skills to solve more of them. Problem solving is a life skill and by developing it, students can create more choices for themselves and the future.

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