AUSTRALIAN MATHEMATICS COMPETITION

AN ACTIVITY OF THE AUSTRALIAN MATHEMATICS TRUST

THURSDAY 5 AUGUST 2010

INTERMEDIATE DIVISION COMPETITION PAPER

AUSTRALIAN SCHOOL YEARS 9 AND 10

TIME ALLOWED: 75 MINUTES

INSTRUCTIONS AND INFORMATION

GENERAL

1. Do not open the booklet until told to do so by your teacher.
2. NO calculators, slide rules, log tables, maths stencils, mobile phones or other calculating aids are permitted. Scribbling paper, graph paper, ruler and compasses are permitted, but are not essential.
3. Diagrams are NOT drawn to scale. They are intended only as aids.
4. There are 25 multiple-choice questions, each with 5 possible answers given and 5 questions that require a whole number answer between 0 and 999. The questions generally get harder as you work through the paper. There is no penalty for an incorrect response.
5. This is a competition not a test; do not expect to answer all questions. You are only competing against your own year in your own State or Region so different years doing the same paper are not compared.
6. Read the instructions on the **Answer Sheet** carefully. Ensure your name, school name and school year are filled in. It is your responsibility that the Answer Sheet is correctly coded.
7. When your teacher gives the signal, begin working on the problems.

THE ANSWER SHEET

1. Use only lead pencil.
2. Record your answers on the reverse of the Answer Sheet (not on the question paper) by FULLY colouring the circle matching your answer.
3. Your Answer Sheet will be read by a machine. The machine will see all markings even if they are in the wrong places, so please be careful not to doodle or write anything extra on the Answer Sheet. If you want to change an answer or remove any marks, use a plastic eraser and be sure to remove all marks and smudges.

INTEGRITY OF THE COMPETITION

The AMC reserves the right to re-examine students before deciding whether to grant official status to their score.

Intermediate Division

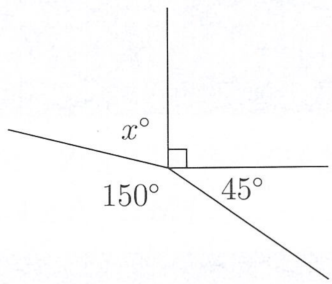
Questions 1 to 10, 3 marks each

1. The value of $\dfrac{3 \times 4}{6}$ is

(A) $\frac{1}{2}$ (B) 1 (C) $\frac{3}{2}$ (D) 2 (E) 3

2. In the diagram, the value of x is

(A) 15 (B) 40 (C) 55 (D) 75 (E) 80



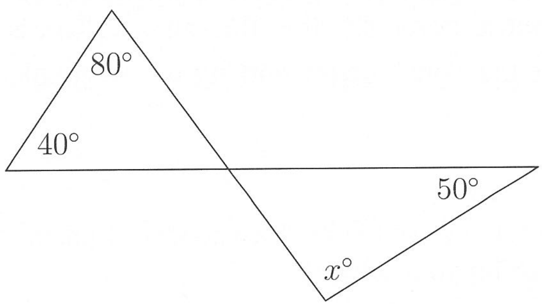
3. The value of $2010 - 20.10$ is

(A) 1990.09 (B) 1990.9 (C) 1989.09 (D) 1989.9 (E) 1998.9

4. If $m = 3$ and $n = -\frac{3}{5}$, then $\frac{m}{n}$ equals

(A) -5 (B) 5 (C) $-\frac{9}{5}$ (D) $-\frac{5}{3}$ (E) 15

5. In the diagram,



the value of x is

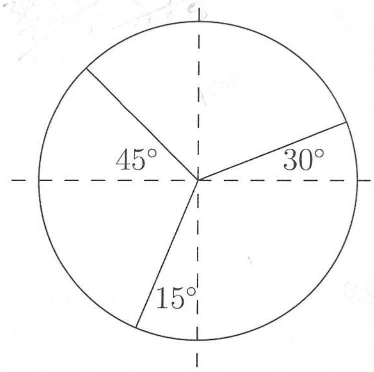
(A) 50 (B) 60 (C) 70 (D) 80 (E) 90

6. Consider all the integers from 1 to 100 inclusive. What is the difference between the sum of all the even numbers and the sum of all the odd numbers?

(A) 0 (B) 25 (C) 50 (D) 100 (E) 200

7. Paul's three children have birthdays in the same week. He bought a circular birthday cake and divided the cake in proportion to their ages as shown. If none of his children is older than 17, what is the sum of their three ages?

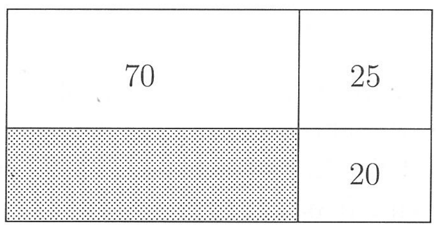
(A) 24 (B) 28 (C) 32 (D) 36 (E) 40



8. What is the remainder when 2^{2010} is divided by 7?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

9. The areas, in square centimetres, of three rectangles are given.



What is the area, in square centimetres, of the shaded rectangle?

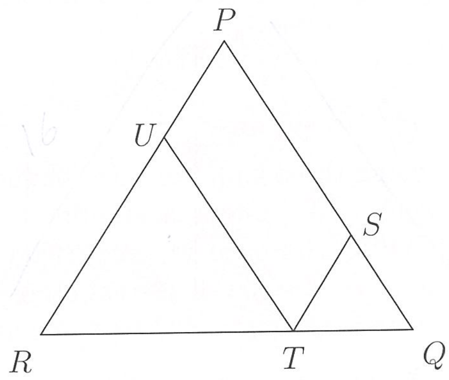
(A) 36 (B) 48 (C) 56 (D) 60 (E) 70

10. If $\frac{1}{6} = \frac{1}{2} + \frac{1}{x}$, the value of x is

(A) -3 (B) $\frac{1}{3}$ (C) 3 (D) 4 (E) -4

Questions 11 to 20, 4 marks each

11. The perimeter of the equilateral triangle PQR is 48 cm.
What is the perimeter, in centimetres, of the parallelogram $PSTU$?



(A) 16 (B) 20 (C) 24 (D) 32 (E) 36

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12. I am going to the shop with $4.20 to spend on my favourite chocolates, hazelnut truffles at 30c each and orange truffles at 50c each. I do not want to buy more than twice as many of one as the other. Apart from that, I want to buy as many truffles as I can. How many is that?

(A) 10 (B) 11 (C) 12 (D) 13 (E) 14

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13. Which of the following numbers cannot be expressed as the sum of two or more consecutive positive integers?

(A) 12 (B) 13 (C) 14 (D) 15 (E) 16

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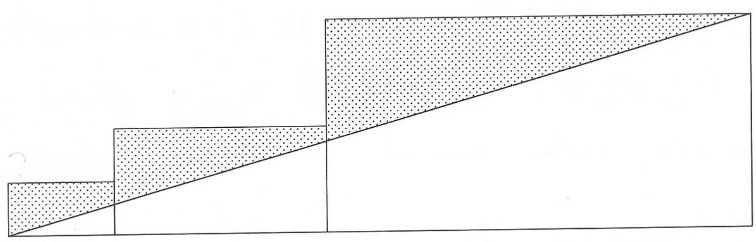
14. Three rectangles are lined up horizontally as shown. The lengths of the rectangles are 2 cm, 4 cm and 8 cm respectively. The heights are 1 cm, 2 cm and 4 cm respectively. A straight line is drawn from the top right-hand corner of the largest rectangle to the bottom left-hand corner of the smallest rectangle.



What is the area, in square centimetres, of the shaded region?

(A) 10 (B) 12 (C) 14 (D) 18 (E) 21

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15. The value of (123456785) × (123456782) − (123456783) × (123456784) is

(A) −2 (B) −1 (C) 0 (D) 1 (E) none of these

16. A three-digit number has all digits odd. How many such numbers are divisible by three?

(A) 29 (B) 36 (C) 39 (D) 40 (E) 41

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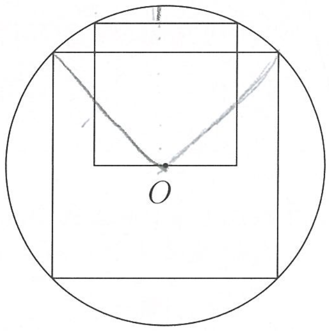
17. Two squares are drawn inside a circle with centre O. The larger square touches the circle at each of its corners. The smaller square touches the circle at two of its corners and one of its sides passes through the centre of the circle as shown.
What is the ratio of the area of the larger square to that of the smaller square?



(A) $3:1$ (B) $5:2$ (C) $\sqrt{5}:2$ (D) $2:1$ (E) $3:\sqrt{6}$

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18. How many integers n from 2 to 10 inclusive have the property that the sum of any consecutive n positive numbers is odd?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

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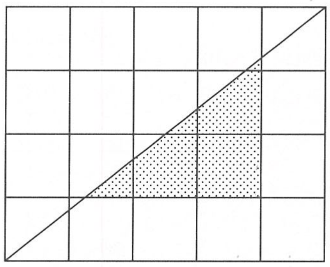
19. The side length of the grid squares in the figure is 1 cm.



What is the area, in square centimetres, of the shaded triangle?

(A) $\frac{120}{40}$ (B) $\frac{111}{40}$ (C) $\frac{116}{40}$ (D) $\frac{125}{40}$ (E) $\frac{121}{40}$

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20. The 5-digit number $a986b$, where a is the first digit and b is the units digit, is divisible by 72. What is the value of $a+b$?

(A) 9 (B) 10 (C) 12 (D) 13 (E) 15

Questions 21 to 25, 5 marks each

21. Snugglepot and Cuddlepie were addicted to the video game *Jabberwocky*. One morning each of them won 70% of their games. That afternoon, they played the same number of games as each other and each won them all. Snugglepot's winning percentage for the day rose to 85% and Cuddlepie's to 90%. The minimum possible total number of games of *Jabberwocky* they could have played that morning was

(A) 10 (B) 20 (C) 30 (D) 60 (E) 70

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22. There is a point X inside a square $PQRS$ such that $PX = 1$, $QX = 2$ and triangles PXQ and PXS are congruent. What is the area, in square units, of the square?

(A) $\frac{1+\sqrt{7}}{2}$ (B) 4 (C) $4-\sqrt{7}$ (D) $4+\sqrt{7}$ (E) 5

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23. In a race, each of four greyhounds runs at its own constant speed. They all start at the same point on a circular track and at 30 seconds into the race, while all are still on their first lap, they have spread out so they are at four corners of a square. How many seconds into the race will it be when they are next at the four corners of a square?

(A) 60 (B) 90 (C) 150 (D) 210 (E) 240

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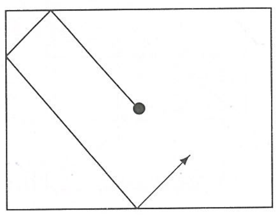
24. A ball starts rolling from the centre of a $3\,\text{m} \times 4\,\text{m}$ room in a direction of 45° to the walls. It rolls in a straight line except when it bounces off a wall and it does that at an angle of 45°.



After it has travelled 20 m, how many times has it bounced off a wall?

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

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25. There are sixteen different ways of writing four-digit strings using 1s and 0s. Three of these strings are 1010, 0100 and 1001. These three can be found as substrings of 101001. There is a string of nineteen 1s and 0s which contains all sixteen strings of length 4 exactly once. If this string starts with 1111, the last four digits are

(A) 1110 (B) 0000 (C) 0110 (D) 1010 (E) 0111

For questions 26 to 30, shade the answer as an integer from 0 to 999 in the space provided on the answer sheet.

Question 26 is 6 marks, question 27 is 7 marks, question 28 is 8 marks, question 29 is 9 marks and question 30 is 10 marks.

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26. How many whole numbers less than 2010 have exactly three factors?

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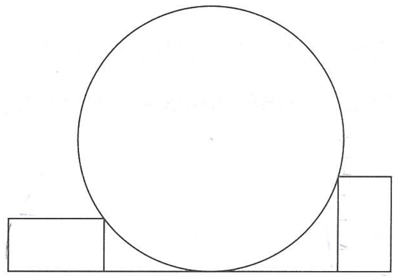
27. Two $10 \times 18 \times \ell$ blocks are placed on either side of a cylinder of length ℓ to stop it from rolling. One block has a $10 \times \ell$ face on the ground while the other block has an $18 \times \ell$ face on the ground. The block on the left sticks out 4 units more than the one on the right.



What is the radius of the cylinder?

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28. A 3-digit number is subtracted from a 4-digit number and the *result* is a 3-digit number.

□□□□ − □□□ = □□□

The 10 digits are all different.
What is the smallest possible result?

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29. I have a list of thirty numbers where the first number is 1, the last number is 30 and each of the other numbers is one more than the average of its two neighbours. What is the largest number in the list?

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30. There are many towns on the island of Tetra, all connected by roads. Each town has three roads leading to three other different towns: one red road, one yellow road and one blue road, where no two roads meet other than at towns. If you start from any town and travel along red and yellow roads alternately (RYRY...) you will get back to your starting town after having travelled over six different roads. In fact RYRYRY will always get you back to where you started. In the same way, going along yellow and blue roads alternately will always get you back to the starting point after travelling along six different roads (YBYBYB). On the other hand, going along red and blue roads alternately will always get you back to the starting point after travelling along four different roads (RBRB). How many towns are there on Tetra?

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