MATHEMATICS LEVEL II

For each of the following problems, decide which is the best of the choices given. Then blacken the corresponding space on the answer sheet.

Notes: (1) Figures that accompany problems in this test are intended to provide information useful in solving the problems. They are drawn as accurately as possible EXCEPT when it is stated in a specific problem that its figure is not drawn to scale. All figures lie in a plane unless otherwise indicated.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

USE THIS SPACE FOR SCRATCHWORK.

1. The set of all ordered pairs (x, y) that satisfy the system $\begin{cases} y = x \\ xy = 1 \end{cases}$ is

   (A) $\{(-1, -1)\}$ (B) $\{(-1, 1)\}$ (C) $\{(1, 1)\}$
   (D) $\{(-1, -1), (1, 1)\}$ (E) $\{(-1, 1), (1, -1)\}$

2. If k is an integer less than zero, which of the following is less than zero?

   (A) $-k$ (B) $-(-k)$ (C) $(-k)^2$ (D) $(k)^2$ (E) $-(k)^3$

3. When a certain integer is divided by 5, the remainder is 3. What is the remainder when 4 times that integer is divided by 5 ?

   (A) 0   (B) 1   (C) 2   (D) 3   (E) 4

4. If $f(x) = -x^2 + 3x + k$ and if $f(-1) = 0$, then $k =$

   (A) 4   (B) 2   (C) 0   (D) -2 (E) -4

5. If functions f, g, and h are defined by $f(x) = 2x$, $g(x) = x + 1$, and $h(x) = x^2$, then $f(g(h(3))) =$

   (A) 14   (B) 16   (C) 18   (D) 20   (E) 22

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6. During the first 2 hours of a 300-mile trip, a car is driven at an average speed of k miles per hour. At what average speed, in miles per hour, must the car be driven for the rest of the distance if the trip takes 4 more hours?

(A) $\frac{k}{2} - 75$ (B) $75 - \frac{k}{2}$ (C) $\frac{1}{75} - \frac{2}{k}$

(D) $\frac{2}{k} - \frac{1}{75}$ (E) $75 - k$

7. If $f(x) = x^2 - x$, then $f(a - 1) =$

(A) $a^2 - a$ (B) $a^2 - a - 1$ (C) $a^2 - a + 1$

(D) $a^2 - a + 2$ (E) $a^2 - 3a + 2$

8. The midpoint of the line segment joining the points (4, 3) and (3, 4) is

(A) $(7, 7)$ (B) $\left(\frac{7}{2}, \frac{7}{2}\right)$ (C) $\left(\frac{5}{2}, \frac{5}{2}\right)$

(D) $\left(2, \frac{3}{2}\right)$ (E) $\left(\frac{1}{2}, -\frac{1}{2}\right)$

9. In Figure 1, if PQ is a diameter of the circle, R is a point on the circle, and $\cos x = \frac{2}{3}$, then $\cos y =$

(A) $\frac{2\sqrt{5}}{25}$ (B) $\frac{\sqrt{5}}{3}$ (C) $\frac{2\sqrt{5}}{5}$ (D) $\frac{3\sqrt{5}}{5}$ (E) $\frac{3\sqrt{5}}{2}$
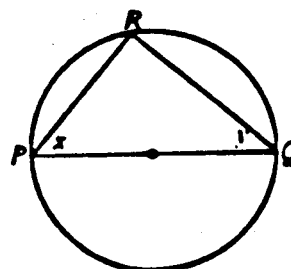

Figure 1

10. The "spread" of a point (x, y) in the rectangular coordinate plane is defined as $|x| + |y|$. Which of the following points has the same spread as $\left(\frac{3}{2}, -\frac{1}{2}\right)$?

(A) $(-1, 0)$ (B) $\left(0, \frac{1}{2}\right)$ (C) $\left(\frac{1}{2}, \frac{1}{2}\right)$

(D) $(1, -1)$ (E) $(2, 1)$

GO ON TO THE NEXT PAGE

11. If a square region is rotated $360°$ around one of its sides as an axis, the solid generated is a

   (A) cube
   (B) rectangular parallelepiped
   (C) cone
   (D) sphere
   (E) cylinder

12. If $f(x, y) = x^2 + xy + y^2$ for all real numbers x and y, which of the following are true?

   I. $f(x, y) = f(x, -y)$
   II. $f(x, y) = f(-x, y)$
   III. $f(x, y) = f(-x, -y)$

   (A) I only   (B) II only   (C) III only

   (D) I and II only   (E) I, II, and III

13. An angle measure of $\frac{\pi}{12}$ radians is equivalent to an angle measure of

   (A) $15°$ (B) $18°$ (C) $30°$ (D) $36°$ (E) $45°$

14. If f is the function defined by $f(x) = 2x - 4$, and if $g(f(x)) = x$, then $g(x) =$

   (A) $\frac{1}{2x-4}$ (B) $-2x + 4$ (C) $x - 2$

   (D) $-\frac{1}{2}x - 2$ (E) $\frac{1}{2}x + 2$

15. The solution set of $\frac{(x+1)^2}{x} > 0$ is

   (A) the empty set   (B) $\{x \mid x > -1\}$ (C) $\{x \mid x > 0\}$
   (D) $\{x \mid x > 1\}$ (E) $\{x \mid x$ is any real number$\}$

16. In Figure 2, the shaded region is bounded by an ellipse whose area A is given by the formula $A = \pi ab$. If the area of the ellipse is 6π and the area of the small circle with center at O is 4π, what is the area of the large circle with center at O?
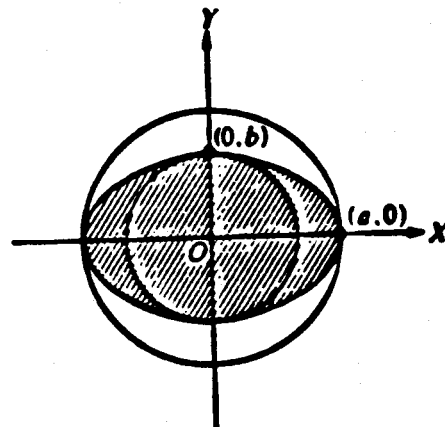
   (A) 5π (B) 6π (C) 7π (D) 8π (E) 9π

(0.b)

(a.0)

Figure 2

17. Where defined, $\dfrac{\sec x}{\csc x} =$

   (A) $\tan x$ (B) $\cot x$ (C) $\sin x \cos x$

   (D) $\dfrac{1}{\sin x \cos x}$ (E) 1

18. In Figure 3, the bases of the right prism are equilateral triangles, each with perimeter 30 centimeters. If the altitude of the prism is 10 centimeters, what is the total surface area of the solid in square centimeters?

   (A) 100 (B) $\dfrac{250}{\sqrt{3}}$ (C) $100\sqrt{3}$
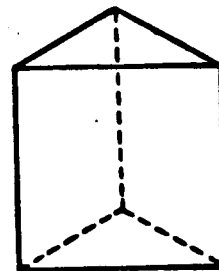
   (D) 300 (E) $50\sqrt{3} + 300$

Figure 3

19. A club has 14 members, consisting of 6 men and 8 women. How many slates of 3 officers—president, vice-president, and secretary—can be formed if the president must be a woman and the vice-president must be a man?

   (A) $2{,}744$ (B) $2{,}184$ (C) 672 (D) 576 (E) 336

20. $\log_2 \sqrt{2} =$

   (A) -1 (B) $-\dfrac{1}{2}$ (C) $\dfrac{1}{2}$ (D) 1 (E) 2

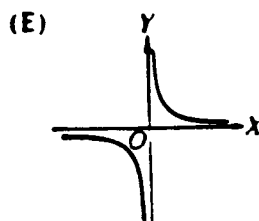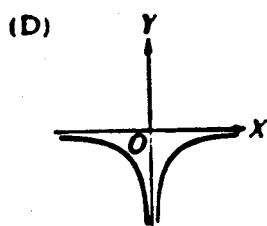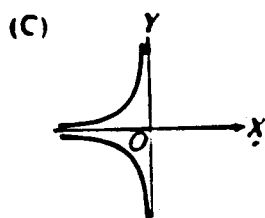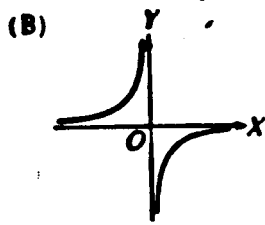21. If $f(x) = \dfrac{x+4}{(x-4)(x^2+4)}$, for what value of x is $f(x)$ undefined?

   (A) -4 (B) -2 (C) 0 (D) 2 (E) 4

GO ON TO THE NEXT PAGE

22. If $f(x) = \frac{1}{x}$, which of the following could be the graph of $y = f(-x)$?

(A)

(B)

(C)

(D)

(E)
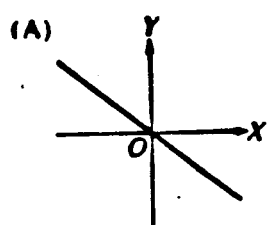
23. If $0 < x < \frac{3\pi}{2}$ and $\cos \frac{\pi}{2} = \sin\left(\frac{\pi}{2} + x\right)$, then $x =$

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) $\frac{3\pi}{4}$ (D) π (E) $\frac{5\pi}{4}$

24. If the line $y = k$ is tangent to the circle $(x - 2)^2 + y^2 = 9$, then $k =$

(A) -1 or 4 (B) -3 or 3 (C) -4 or 1
(D) -6 or 6 (E) -9 or 9

25. If, for all x, $3^x + 3^x + 3^x = k3^{x+1}$, then $k =$

(A) 9^{3x} (B) 3^{x^2-x-1} (C) 3^{2x-1} (D) 3 (E) 1

GO ON TO THE NEXT PAGE

26. If $f(x) = (x + 3)^2 + 1$, what is the minimum value of the function f?

    (A) −3   (B) 0   (C) 1   (D) 3   (E) 4

27. In Figure 4, if $\text{Arcsin } x = \text{Arccos } x$, then $k =$

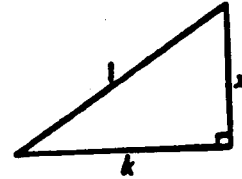   (A) x (B) x^2 (C) 1   (D) $1 - x$ (E) $\frac{1}{x}$
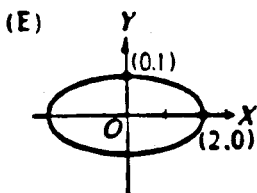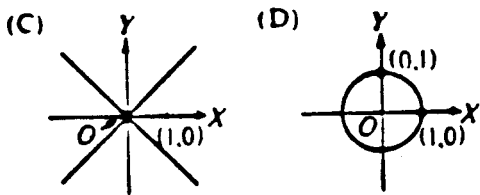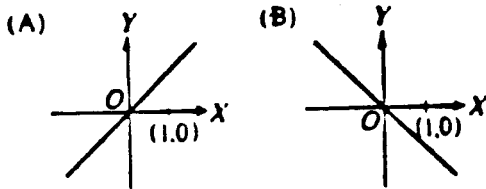

Figure 4

<u>Note:</u> Figure not drawn to scale.

28. Which of the following could be the graph of the set of all pairs (x, y), where $x = \cos\theta$, $y = \sin\theta$, and $0 \leq \theta < 2\pi$?

(A)

(B)

(C)

(D)

(E)

29. If $x + 2$ is a factor of $2x^3 + x^2 - 2kx + 4$, then k is

    (A) −6   (B) −4   (C) 2   (D) 4   (E) 6

GO ON TO THE NEXT PAGE

USE THIS SPACE FOR SCRATCHWORK.

30. If $f(x) = x^2 + x - 6$, then the set of all b for which $f(-b) = f(b)$ is

(A) all real numbers   (B) $\{-3, 2\}$ (C) $\{-2, 3\}$

(D) $\{0\}$ (E) $\{2\}$

31. If $\sin x = -\cos x$ and $0 \leqslant x \leqslant \pi$, then $x =$

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) $\frac{2\pi}{3}$ (E) $\frac{3\pi}{4}$

32. Which of the following could be an equation of the graph shown in Figure 5?

(A) $y = \sin x + 1$ (B) $y = \cos x - 1$ (C) $y = \csc x - 1$

(D) $y = \sec x - 1$ (E) $y = \csc x + 1$
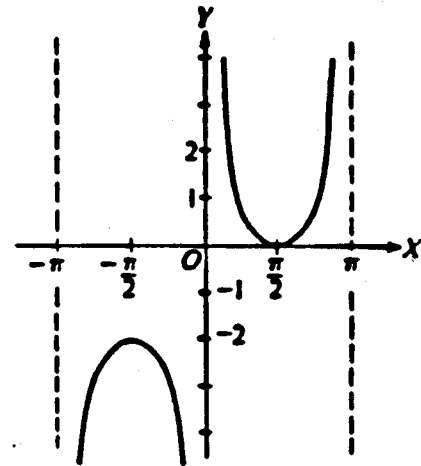
Figure 5

33. $(-i)^n$ is a negative real number if $n =$

(A) 21   (B) 22   (C) 23   (D) 24   (E) 25

34. If, for all x, $f(x) = a^x$ and $f(x + 3) = 8f(x)$, then $a =$

(A) 0   (B) 1   (C) 2   (D) 4   (E) 8

35. If $\sin x = \frac{1}{2}$ and $0 \leqslant x \leqslant \frac{\pi}{2}$, then $\sin 2x =$

(A) $-\frac{\sqrt{3}}{2}$ (B) $-\frac{1}{2}$ (C) 0   (D) $\frac{\sqrt{3}}{2}$ (E) 1

36. $\dfrac{(n - 1)!}{n!} + \dfrac{(n + 1)!}{n!} =$

(A) $\frac{n - 1}{n}$ (B) $\frac{n^2 + 1}{n}$ (C) $\frac{n^2 - 1}{n}$

(D) $\frac{n + 1}{n}$ (E) $\frac{n^2 + n + 1}{n}$

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USE THIS SPACE FOR SCRATCHWORK.

37. What is the range of the function defined by

$f(x) = \frac{1}{x} + 2$?

(A) All real numbers

(B) All real numbers except $-\frac{1}{2}$

(C) All real numbers except 0

(D) All real numbers except 2

(E) All real numbers between 2 and 3

38. If $a > b$ and $c > d$, which of the following must be true?

    I. $a + c > b + d$
    II. $ac > bd$
    III. $a > d$

(A) I only   (B) II only   (C) I and II only
(D) I and III only   (E) II and III only

39. If $\sum_{k=0}^{10} (3 + k) = X + \sum_{k=0}^{10} k$, then $X =$

(A) 3   (B) 10   (C) 11   (D) 30   (E) 33

40. How many different sets of two parallel edges are there in a cube?

(A) 6   (B) 8   (C) 12   (D) 18   (E) 24

41. Which of the following defines a function that will associate a positive integer y with each positive integer x so that x and y have the same tens' digit?

(A) $y = 10x$ (B) $y = 11x$ (C) $y = 100x$
(D) $y = 101x$ (E) $y = 111x$

42. If two fair dice are tossed, what is the probability that the sum of the number of dots on the top faces will be 10?

(A) $\frac{1}{36}$ (B) $\frac{1}{18}$ (C) $\frac{1}{12}$ (D) $\frac{1}{9}$ (E) $\frac{1}{6}$

43. Three vertices of a cube, no two of which lie on the same edge, are joined to form a triangle. If an edge of the cube has length 1, what is the area of the triangle?

   (A) $\dfrac{\sqrt{6}}{2}$ (B) $\dfrac{\sqrt{3}}{2}$ (C) $\dfrac{\sqrt{2}}{2}$ (D) $\dfrac{\sqrt{6}}{4}$ (E) $\dfrac{\sqrt{3}}{4}$

44. What is $\displaystyle\lim_{x\to2}\dfrac{x^3 + x^2 - 6x}{x - 2}$?

   (A) 0   (B) 3   (C) 7   (D) 10

   (E) The limit does not exist.

45. Which of the following graphs could represent the equation $y = ax^2 + bx + c$ where $b^2 - 4ac > 0$?
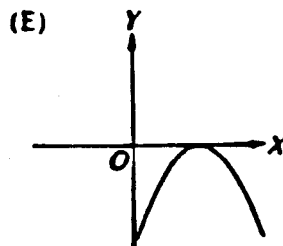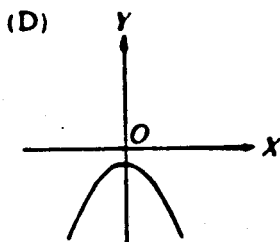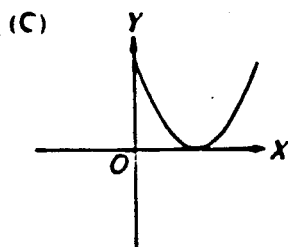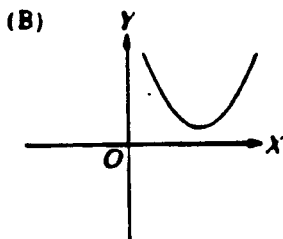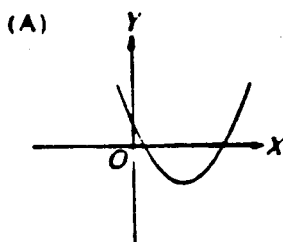
   (A)

   (B)

   (C)

   (D)

   (E)

46. The least positive integer N for which each of

$\frac{N}{2} \cdot \frac{N}{3} \cdot \frac{N}{4} \cdot \frac{N}{5} \cdot \frac{N}{6} \cdot \frac{N}{7} \cdot \frac{N}{8}$, and $\frac{N}{9}$ is an integer is

(A) $9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2$

(B) $9 \cdot 8 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2$

(C) $9 \cdot 8 \cdot 7 \cdot 6 \cdot 5$

(D) $9 \cdot 8 \cdot 7 \cdot 5$

(E) $9 \cdot 8 \cdot 7$

47. The graph of $y = 3 + \cos 2x$ intersects the Y-axis at the point where $y =$

(A) 0   (B) 1   (C) 3   (D) 4   (E) 5

48. The area of the parallelogram in Figure 6 is

(A) ab (B) $ab \cos \theta$ (C) $ab \sin \theta$

(D) $ab \tan \theta$ (E) $a^2 + b^2 - 2ab \cos \theta$
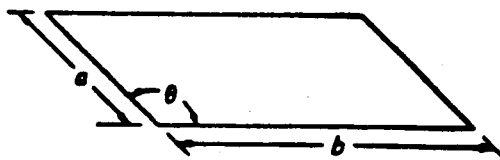
Figure 6

49. Which of the following equations describes the set of all points (x, y) that are equidistant from the X-axis and the point $(4, 6)$?

(A) $(x - 4)^2 + (y - 6)^2 = 9$

(B) $(x - 4)^2 = 12(y - 3)$

(C) $(y - 3)^2 = 12(x - 4)$

(D) $(x - 4)^2 = 6(y - 3)$

(E) $(x - 4)^2 = 12(y - 6)$

50. "If A is true, then for some x, B is true." Which of the following is logically equivalent to the preceding statement?

(A) If B is false for all x, then A is false.
(B) If for some x, B is true, then A is true.
(C) If A is false, then for all x, B is false.
(D) If B is false for some x, then A is false.
(E) There exists an x for which A is true and B is false.

S · T O P

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON THIS TEST ONLY.
DO NOT WORK ON ANY OTHER TEST IN THIS BOOK.

TABLE A

Answers to Mathematics Test, Level II, Form 3EAC2, and Percentage of Students Answering Each Question Correctly

Question Number	Correct Answer	Right	Wrong	Percentage of Students Answering the Question Correctly
1	D			92%
2	B			95
3	C			79
4	A			82
5	D			95
6	B			56
7	E			89
8	B			93
9	B			59
10	D			84
11	E			75
12	C			90
13	A			91
14	E			73
15	C			78
16	E			80
17	A			79
18	E			74
19	D			31
20	C			67
21	E			91
22	B			75
23	B			67
24	B			65
25	E			56
26	C			67
27	A			65
28	D			76
29	C			52
30	D			72
31	E			69
32	C			57
33	B			42
34	C			42
35	D			69
36	E			40
37	D			36
38	A			52
39	E			27
40	D			35
41	D			38
42	C			42
43	B			26
44	D			47
45	A			25
46	D			39
47	D			41
48	C			21
49	B			36
50	A			36

Note: The percentages are based on the analysis of the answer sheets for a random sample of students who took this test in November 1982 and whose mean score was 675.

285

Finding Your College Board Scaled Score ▬

When you take Achievement Tests, the scores sent to the colleges you specify will be reported on the College Board scale, ranging from 200 to 800. The raw score that you obtained above (Step 4) can be converted to a scaled score by using Table B.

To find your scaled score on this test, locate your raw score in the left column of Table B; the corresponding score in the right column will be your College Board scaled score. For example, a raw score of 15 on this particular edition of the Mathematics Achievement Test, Level II, corresponds to a College Board scaled score of 570. Raw scores are converted to scaled scores to ensure that a score earned on any one edition of the Mathematics Achievement Test, Level II, is comparable to the same scaled score earned on any other edition of the test.

Because some editions of the Mathematics Achievement Test, Level II, may be slightly easier or more difficult than others, statistical adjustments are made in the scores so that each College Board scaled score indicates the same level of performance, regardless of the edition of the test you take and the ability of the group you take it with. A given raw score will correspond to different College Board scores, depending on the edition of the test taken. A raw score of 40, for example, may convert to a College Board score of 780 on one edition of the test, but that raw score might convert to a College Board score of 800 on a slightly more difficult edition. When you take the Mathematics Achievement Test, Level II, on the actual test day, your score is likely to differ somewhat from the score you obtained on this test. People perform at different levels at different times, for reasons unrelated to the test itself. The precision of any test is also limited because it represents only a sample of all the possible questions that could be asked.(See page 12, "How Precise Are Your Scores?" for further information.)

Reviewing Your Test Performance

After you have scored your test, you should take some time to consider the following points in relation to your performance on the test.

- *Did you run out of time before you reached the end of the test?*
  If you did, you may want to consider tactics that will help you pace yourself better. For example, you may have spent too much time working on one or two difficult questions. A better approach might have been to continue the test and return to those questions after you had attempted to answer the remaining questions on the test.

- *Did you take a long time reading the directions for the test?*
  The directions in this test are the same as those in the Mathematics Achievement Tests, Level II now being administered. You will save time when you read the directions on the test day if you become thoroughly familiar with them in advance.

- *How did you handle questions you were unsure of?*
  If you were able to eliminate one or more of the answer choices and you guessed from the remaining choices, then your approach probably worked to your advantage. On the other hand, omitting questions about which you have some knowledge or guessing answers haphazardly would probably be a mistake.

- *How difficult were the questions for you compared with other students who took the test?*
  By referring to Table A on page 285 you can find out how difficult each question was for the group

TABLE B — SCORE CONVERSION TABLE
Mathematics Achievement Test, Level II, Form 3EAC2

Raw Score	College Board Scaled Score	Raw Score	College Board Scaled Score
50	800	20	610
49	800	19	610
48	800	18	600
47	800	17	590
46	800	16	580
45	800	15	570
44	800	14	560
43	800	13	560
42	800	12	550
41	790	11	530
40	780	10	520
39	770	9	500
38	760	8	480
37	760	7	470
36	750	6	450
35	740	5	430
34	730	4	420
33	720	3	400
32	710	2	380
31	710	1	370
30	700	0	350
29	690	−1	330
28	680	−2	320
27	670	−3	300
26	660	−4	280
25	660	−5	270
24	650	−6	250
23	640	−7	230
22	630	−8	220
21	620	−9 through −12	200

Mathematics Achievement Test, Level II

The test that follows is an edition of the Mathematics Achievement Test, Level II, administered in November 1982. So that you will have an idea of what the actual test administration will be like, try to take this test under conditions as close as possible to those of the actual test. It will probably help if you

- Set aside an hour for the test when you will not be interrupted, so that you can complete all of it in one sitting.

- Sit at a desk with no other papers or books. You can't take a calculator, a dictionary, other books, or notes into the test room.

- Have a kitchen timer or clock in front of you for timing yourself.

- Tear out an answer sheet from the back of this book and fill it in just as you would on the day of the test. You can use one answer sheet for as many as three Achievement Tests.

- Read the instructions that precede the test. When you take the test, you will be asked to read them before you begin answering questions.

- After you finish the test, read the sections on "How to Score the Mathematics Achievement Test, Level II," and "Reviewing Your Test Performance," which follow the test.