AUSTRALIAN MATHEMATICS COMPETITION

AN ACTIVITY OF THE AUSTRALIAN MATHEMATICS TRUST

THURSDAY 6 AUGUST 2009

SENIOR DIVISION COMPETITION PAPER

AUSTRALIAN SCHOOL YEARS 11 AND 12

TIME ALLOWED: 75 MINUTES

INSTRUCTIONS AND INFORMATION

GENERAL

1. Do not open the booklet until told to do so by your teacher.
2. NO calculators, slide rules, log tables, maths stencils, mobile phones or other calculating aids are permitted. Scribbling paper, graph paper, ruler and compasses are permitted, but are not essential.
3. Diagrams are NOT drawn to scale. They are intended only as aids.
4. There are 25 multiple-choice questions, each with 5 possible answers given and 5 questions that require a whole number answer between 0 and 999. The questions generally get harder as you work through the paper. There is no penalty for an incorrect response.
5. This is a competition not a test; do not expect to answer all questions. You are only competing against your own year in your own State or Region so different years doing the same paper are not compared.
6. Read the instructions on the **Answer Sheet** carefully. Ensure your name, school name and school year are filled in. It is your responsibility that the Answer Sheet is correctly coded.
7. When your teacher gives the signal, begin working on the problems.

THE ANSWER SHEET

1. Use only lead pencil.
2. Record your answers on the reverse of the Answer Sheet (not on the question paper) by FULLY colouring the circle matching your answer.
3. Your Answer Sheet will be read by a machine. The machine will see all markings even if they are in the wrong places, so please be careful not to doodle or write anything extra on the Answer Sheet. If you want to change an answer or remove any marks, use a plastic eraser and be sure to remove all marks and smudges.

INTEGRITY OF THE COMPETITION

The AMC reserves the right to re-examine students before deciding whether to grant official status to their score.

Senior Division

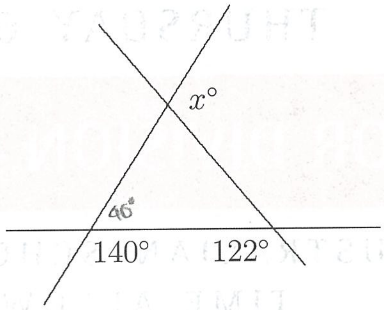
Questions 1 to 10, 3 marks each

1. The value of $(2009 + 9) - (2009 - 9)$ is

(A) 4000 (B) 2018 (C) 3982 (D) 0 (E) 18

2. In the diagram, x equals

(A) 140 (B) 122 (C) 80 (D) 90 (E) 98



3. The graph of $y = kx$ passes through the point $(-2, -1)$. The value of k is

(A) 2 (B) -2 (C) 4 (D) $\frac{1}{2}$ (E) $-\frac{1}{2}$

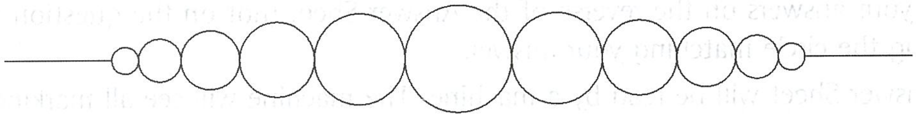
4. The value of $(0.6)^{-2}$ is

(A) -0.36 (B) 0.036 (C) $\frac{9}{25}$ (D) $\frac{25}{9}$ (E) 3.6

5. $(x - y) - 2(y - z) + 3(z - x)$ equals

(A) $-2x - 3y + 5z$ (B) $-2x - 3y - z$ (C) $4x + y - z$ (D) $4x + 3y - z$ (E) $2x + 3y - 5z$

6. On a string of beads, the largest bead is in the centre and the smallest beads are on the ends. The size of the beads increases from the ends to the centre as shown in the diagram.



The smallest beads cost \$1 each, the next smallest beads cost \$2 each, the next smallest \$3 each, and so on. How much change from \$200 would there be for the beads on a string with 25 such beads?

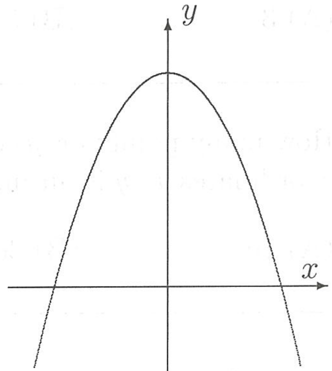
(A) \$25 (B) \$31 (C) \$40 (D) \$52 (E) \$55

7. If $a * b = a + \frac{1}{b}$ for every pair a, b of positive numbers, the value of $1 * (2 * 3)$ is

(A) $\frac{10}{3}$ (B) $\frac{10}{7}$ (C) $\frac{11}{6}$ (D) $\frac{9}{2}$ (E) $\frac{3}{10}$

---

8. The graph of $y = ax^2 + bx + c$ is shown, with its vertex on the y-axis. Which of the following statements must be true?



(A) $a + b + c = 0$ (B) $a + b - c < 0$

(C) $-a + b - c > 0$ (D) $a + b + c < 0$

(E) there is not enough information

---

9. In a school of 1000 students, 570 are girls. One-quarter of the students travel to school by bus and 313 boys do **not** go by bus. How many girls travel to school by bus?

(A) 7 (B) 63 (C) 153 (D) 180 (E) 133

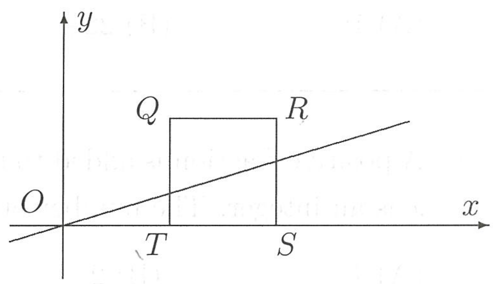
---

10. A box in the dressing shed of a sporting team contains 6 green and 3 red caps. The probability that the first 2 caps taken at random from the box will be the same colour is

(A) $\frac{1}{2}$ (B) $\frac{5}{12}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$ (E) $\frac{2}{9}$

Questions 11 to 20, 4 marks each

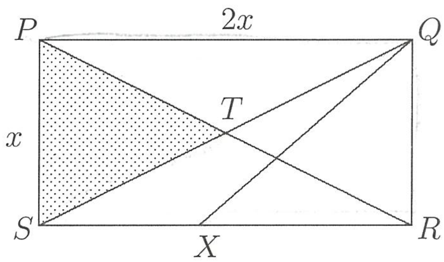
11. $QRST$ is a square with T at (1, 0) and S at (2, 0). Which of the following is an equation of the line through the origin which bisects the area of the square?



(A) $y = \frac{1}{2}x$ (B) $y = \frac{1}{3}x$ (C) $y = \frac{2}{3}x$
(D) $y = 2x$ (E) $y = 3x$

---

12. A rectangle $PQRS$ has $PQ = 2x$ cm and $PS = x$ cm. The diagonals PR and QS meet at T. X lies on RS so that QX divides the pentagon $PQRST$ into two sections of equal area. The length, in centimetres, of RX is



(A) $\frac{x}{2}$ (B) x (C) $\frac{5x}{4}$ (D) $\frac{3x}{2}$ (E) $\frac{3x}{4}$

13. The solution to the equation $5^x - 5^{x-2} = 120\sqrt{5}$ is a rational number of the form $\frac{a}{b}$, where $b \neq 0$ and a and b are positive and have no common factors. What is the value of $a+b$?

(A) 3 (B) 5 (C) 7 (D) 9 (E) 11

---

14. How many points (x, y) on the circle $x^2 + y^2 = 50$ are such that at least one of the coordinates x, y is an integer?

(A) 16 (B) 30 (C) 48 (D) 60 (E) 100

---

15. An *eyebrow* is an arrangement of the numbers $1, 2, 3, 4$ and 5 such that the second and fourth numbers are each bigger than both their immediate neighbours. For example, $(1, 3, 2, 5, 4)$ is an eyebrow and $(1, 3, 4, 5, 2)$ is not.
The number of eyebrows is

(A) 16 (B) 12 (C) 15 (D) 24 (E) 18

---

16. The sum of the positive solutions to the equation $(x^2 - x)^2 = 18(x^2 - x) - 72$ is

(A) 5 (B) 7 (C) 8 (D) 9 (E) 18

---

17. On a clock face, what is the size, in degrees, of the acute angle between the line joining the 5 and the 9 and the line joining the 3 and the 8?

(A) 15 (B) $22\frac{1}{2}$ (C) 30 (D) 45 (E) 60

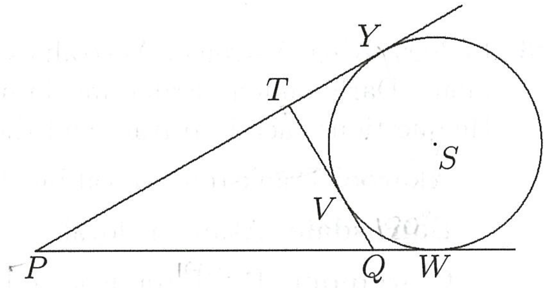
---

18. A positive fraction is added to its reciprocal. The sum is $\frac{x}{60}$ in lowest terms, where x is an integer. The number of possible values of x is

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

---

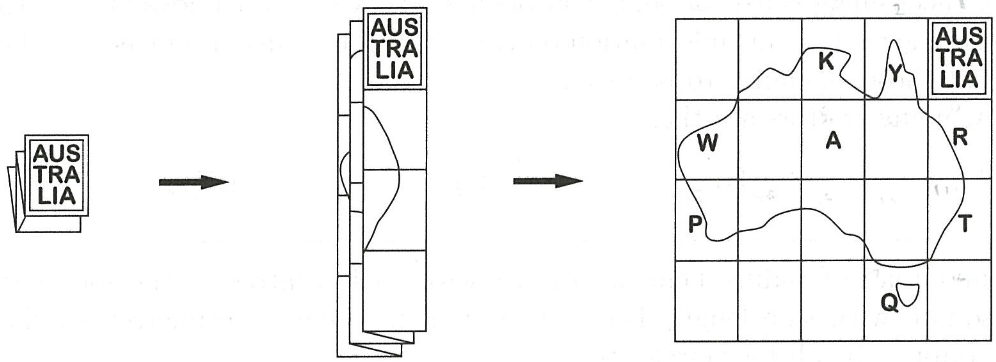
19. In $\triangle PQT$, $PQ = 10\,\text{cm}$, $QT = 5\,\text{cm}$ and $\angle PQT = 60°$.
PW, PY and TQ are tangents to the circle with centre S at W, Y and V respectively.
The radius of the circle, in centimetres, is



(A) $\dfrac{5\sqrt{3}}{2+\sqrt{3}}$ (B) $\dfrac{5(3-\sqrt{3})}{2}$ (C) $\dfrac{5}{1+\sqrt{3}}$ (D) $\dfrac{5\sqrt{3}}{2}$ (E) $\dfrac{25\sqrt{3}}{6}$

---

20. I bought a map of Australia, unfolded it and marked eight places I wanted to visit.



I then refolded the map and placed it back on the table as it was. In what order are my marks stacked from top to bottom?

(A) RTYQKAWP (B) YKRAWTPQ (C) RTQYKAWP (D) YKTPRAWQ (E) YKWARTPQ

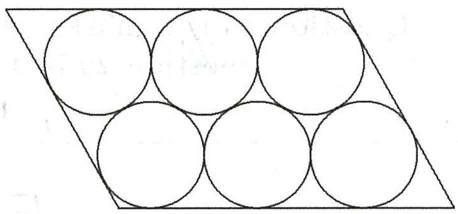
Questions 21 to 25, 5 marks each

21. A palindromic number is a 'symmetrical' number which reads the same forwards as backwards. For example, 55, 101 and 8668 are palindromic numbers.
There are 90 four-digit palindromic numbers.
How many of these four-digit palindromic numbers are divisible by 7?

(A) 7 (B) 9 (C) 14 (D) 18 (E) 21

---

22. What is the area, in square centimetres, of the parallelogram that would fit snugly around 6 circles, each of radius 3 cm, as shown in the diagram?



(A) 108 (B) $8(4+3\sqrt{3})$ (C) $15(2+\sqrt{3})$ (D) $12(9+5\sqrt{3})$ (E) 216

23. In 3009, King Warren of Australia suspects the Earls of Akaroa, Bairnsdale, Claremont, Darlinghurst, Erina and Frankston are plotting a conspiracy against him. He questions each in private and they tell him:

Akaroa: Frankston is loyal but Erina is a traitor.

Bairnsdale: Akaroa is loyal.

Claremont: Frankston is loyal but Bairnsdale is a traitor.

Darlinghurst: Claremont is loyal but Bairnsdale is a traitor.

Erina: Darlinghurst is a traitor.

Frankston: Akaroa is loyal.

Each traitor knows who the other traitors are, but will always give false information, accusing loyalists of being traitors and vice versa. Each loyalist tells the truth as he knows it, so his information on traitors can be trusted, but he may be wrong about those he claims to be loyal.
How many traitors are there?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

---

24. Four circles of radius 1 cm are drawn with their centres at the four vertices of a square with side length 1 cm. The area, in square centimetres, of the region overlapped by all four circles is

(A) $2\sqrt{3}-\pi$ (B) $\pi-\sqrt{2}$ (C) $1+\frac{\pi}{3}-\sqrt{3}$

(D) $\pi-2\sqrt{2}$ (E) $\frac{\pi-3-\sqrt{3}}{2}$

---

25. Let $f(x)=\frac{x+6}{x}$ and $f_n(x)=f(f(\cdots(f(x))\cdots))$ be the n-fold composite of f.
For example, $f_2(x)=\frac{\frac{x+6}{x}+6}{\frac{x+6}{x}}=\frac{7x+6}{x+6}$ and $f_3(x)=\frac{\frac{7x+6}{x+6}+6}{\frac{7x+6}{x+6}}=\frac{13x+42}{7x+6}$.
Let S be the complete set of real solutions of the equation $f_n(x)=x$. The number of elements in S is

(A) 2 (B) $2n$ (C) 2^n (D) 1 (E) infinite

For questions 26 to 30, shade the answer as an integer from 0 to 999 in the space provided on the answer sheet.

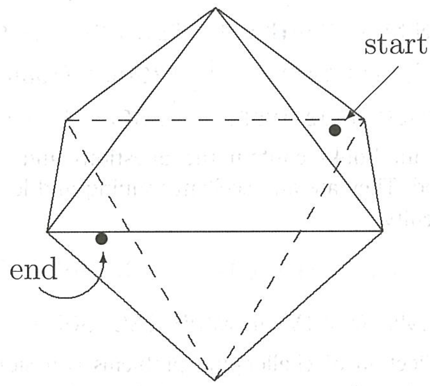
Question 26 is 6 marks, question 27 is 7 marks, question 28 is 8 marks, question 29 is 9 marks and question 30 is 10 marks.

---

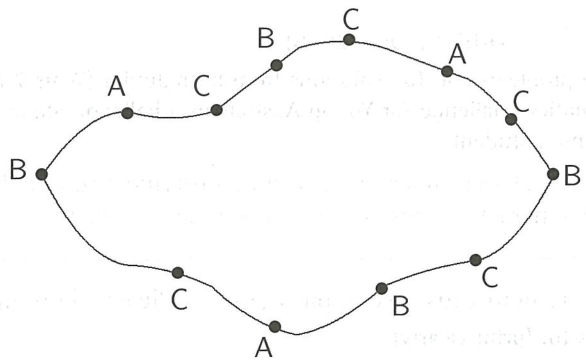
26. The reciprocals of 4 positive integers add up to $\frac{19}{20}$. Three of these integers are in the ratio $1:2:3$. What is the sum of the four integers?

27. We say a number is *ascending* if its digits are strictly increasing. For example, 189 and 3468 are ascending while 142 and 466 are not. For which ascending 3-digit number n (between 100 and 999) is $6n$ also ascending?

28. A regular octahedron has edges of length 6 cm. If d cm is the shortest distance from the centre of one face to the centre of the opposite face measured around the surface of the octahedron, what is the value of d^2?



29. The country of Big Wally has a railway which runs in a loop 1080 km long. Three companies, A, B and C run trains on the track and plan to build stations. Company A will build three stations, equally spaced at 360 km intervals. Company B will build four stations at 270 km intervals and Company C will build five stations at 216 km intervals.



The government tells them to space their stations so that the longest distance between consecutive stations is as small as possible. What is this distance in kilometres?

30. A trapezium $ABCD$ has $AD \parallel BC$ and a point E is chosen on the base AD so that the line segments BE and CE divide the trapezium into three right-angled triangles. These three triangles are similar, but no two are congruent. In common units, all the triangles' side lengths are integers. The length of AD is 2009. What is the length of BC?

A SELECTION OF AUSTRALIAN MATHEMATICS TRUST PUBLICATIONS

Indicate Quantity Required in Box

AUSTRALIAN MATHEMATICS COMPETITION BOOKS

☐ **2009 AMC SOLUTIONS AND STATISTICS SECONDARY VERSION – $A35.00 EACH**

☐ **2009 AMC SOLUTIONS AND STATISTICS PRIMARY AND SECONDARY VERSIONS – $A57.00 FOR BOTH**

Two books are published each year for the Australian Mathematics Competition, a Primary version for the Middle and Upper Primary divisions and a Secondary version for the Junior, Intermediate and Senior divisions. The books include the questions, full solutions, prize winners, statistics, information on Australian achievement rates, analyses of the statistics as well as discrimination and difficulty factors for each question. The 2009 books will be available early 2010.

AUSTRALIAN MATHEMATICS COMPETITION – $A40.00 EACH

☐ **BOOK 1 (1978–1984)** ☐ **BOOK 2 (1985–1991)** ☐ **BOOK 3 (1992–1998)**

☐ **BOOK 3-CD (1992–1998)** ☐ **BOOK 4 (1999–2005)**

These four books contain the questions and solutions from the Australian Mathematics Competition for the years indicated. They are an excellent training and learning resource with questions grouped into topics and ranked in order of difficulty.

BOOKS FOR FURTHER DEVELOPMENT OF MATHEMATICAL SKILLS

☐ **PROBLEMS TO SOLVE IN MIDDLE SCHOOL MATHEMATICS – $A50.00 EACH**

This collection of challenging problems is designed for use with students in Years 5 to 8. Each of the 65 problems is presented ready to be photocopied for classroom use. With each problem there are teacher's notes and fully worked solutions. Some problems have extension problems presented with the teacher's notes. The problems are arranged in topics (Number, Counting, Space and Number, Space, Measurement, Time, Logic) and are roughly in order of difficulty within each topic.

☐ **PROBLEM SOLVING VIA THE AMC – $A40.00 EACH**

This book uses nearly 150 problems from past AMC papers to demonstrate strategies and techniques for problem solving. The topics selected include Geometry, Motion and Counting Techniques.

CHALLENGE! – $A40.00 EACH

☐ **BOOK 1 (1991–1998)** ☐ **BOOK 2 (1999–2006)**

These books reproduce the problems and full solutions from both Junior (Years 7 and 8) and Intermediate (Years 9 and 10) versions of the Mathematics Challenge for Young Australians, Challenge Stage. They are valuable resource books for the classroom and the talented student.

The above prices are current to 31 December 2009. Online ordering and details of other AMT publications are available on the Australian Mathematics Trust's web site www.amt.edu.au

PAYMENT DETAILS

Payment must accompany orders. Please allow up to 14 days for delivery.

Please forward publications to: (print clearly)

Name: ______________________________

Address: ______________________________

____

Country: ______________ Postcode: ______________

POSTAGE AND HANDLING – within Australia, add $A4.00 for the first book and $A2.00 for each additional book
– outside Australia, add $A13.00 for the first book and $A5.00 for each additional book

Cheque/Bankdraft enclosed for the amount of $A ______________

Please charge my Credit Card (Visa, Mastercard) Amount authorised:$A ______________ Date: / /

Cardholder's Name (as shown on card): ______________

Cardholder's Signature: ______________ Tel (bh): ______________

Card Number: ☐☐☐☐ ☐☐☐☐ ☐☐☐☐ ☐☐☐☐ Expiry Date: /

All payments (cheques/bankdrafts, etc.) should be in Australian currency and be made payable to **AUSTRALIAN MATHEMATICS TRUST** and sent to:

Australian Mathematics Trust, University of Canberra ACT 2601, Australia.
Tel: 02 6201 5137 Fax: 02 6201 5052