AUSTRALIAN MATHS TRUST

AUSTRALIAN MATHEMATICS COMPETITION

Senior Years 11 & 12

(Australian school years)

THURSDAY 1 AUGUST 2019

NAME: ____________________

TIME ALLOWED: 75 minutes

INSTRUCTIONS AND INFORMATION

General

1 Do not open the booklet until told to do so by your teacher.

2 NO calculators, maths stencils, mobile phones or other calculating aids are permitted. Scribbling paper, graph paper, ruler and compasses are permitted, but are not essential.

3 Diagrams are NOT drawn to scale. They are intended only as aids.

4 There are 25 multiple-choice questions, each requiring a single answer, and 5 questions that require a whole number answer between 0 and 999. The questions generally get harder as you work through the paper. There is no penalty for an incorrect response.

5 This is a competition not a test; do not expect to answer all questions. You are only competing against your own year in your own country/Australian state so different years doing the same paper are not compared.

6 Read the instructions on the answer sheet carefully. Ensure your name, school name and school year are entered. It is your responsibility to correctly code your answer sheet.

7 When your teacher gives the signal, begin working on the problems.

The answer sheet

1 Use only lead pencil.

2 Record your answers on the reverse of the answer sheet (not on the question paper) by FULLY colouring the circle matching your answer.

3 Your answer sheet will be scanned. The optical scanner will attempt to read all markings even if they are in the wrong places, so please be careful not to doodle or write anything extra on the answer sheet. If you want to change an answer or remove any marks, use a plastic eraser and be sure to remove all marks and smudges.

Integrity of the competition

The AMT reserves the right to re-examine students before deciding whether to grant official status to their score.

Reminder: You may sit this competition once, in one division only, or risk no score.

Senior Division

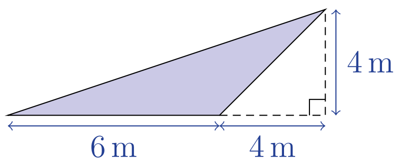
Questions 1 to 10, 3 marks each

1. What is the value of 201×9?

(A) 189 (B) 1809 (C) 1818 (D) 2001 (E) 2019

2. What is the area of the shaded triangle?

(A) $8\,\mathrm{m}^2$ (B) $12\,\mathrm{m}^2$ (C) $14\,\mathrm{m}^2$ (D) $20\,\mathrm{m}^2$ (E) $24\,\mathrm{m}^2$

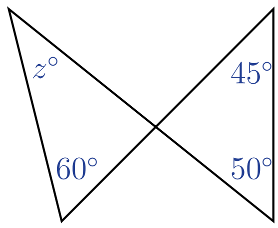


3. What is 19% of \$20?

(A) \$20.19 (B) \$1.90 (C) \$0.19 (D) \$3.80 (E) \$0.38

4. What is the value of z?

(A) 30 (B) 35 (C) 45 (D) 50 (E) 55



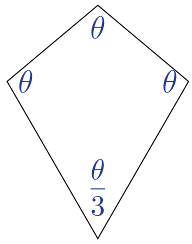
5. The value of $2^0 + 1^9$ is

(A) 1 (B) 2 (C) 3 (D) 10 (E) 11

6. Let $f(x) = 3x^2 - 2x$. Then $f(-2) =$

(A) -32 (B) -8 (C) 16 (D) 32 (E) 40

7. This kite has angles θ, θ, θ and $\dfrac{\theta}{3}$.
What is the size of the angle θ?



(A) 120° (B) 105° (C) 90° (D) 112° (E) 108°

---

8. Consider the *undulating* number sequence

$$1, 4, 7, 4, 1, 4, 7, 4, 1, 4, \ldots\ ,$$

which repeats every four terms. The running total of the first 3 terms is 12. The running total of the first 7 terms is 28.
Which one of the following is also a running total of this sequence?

(A) 61 (B) 62 (C) 67 (D) 66 (E) 65

---

9. Mia walks at 1.5 metres per second. Her friend Crystal walks at 2 metres per second. They walk in opposite directions around their favourite bush track, starting together from the same point. They first meet again after 20 minutes.
How long, in kilometres, is the track?

(A) 3.5 (B) 4.2 (C) 6 (D) 7 (E) 8.4

---

10. $\dfrac{1^1 + 2^2 + 3^3 + 4^4}{1^1 + 2^2 + 3^3} =$

(A) 2^3 (B) 3^2 (C) 11 (D) 4^3 (E) 259

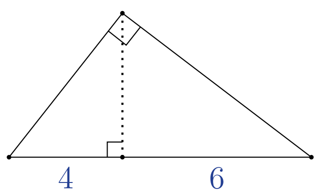
Questions 11 to 20, 4 marks each

11. The 5-digit number $P679Q$ is divisible by 72. The digit P is equal to

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

---

12. The altitude of a right-angled triangle divides the hypotenuse into lengths of 4 and 6. What is the area of the triangle?



(A) $10\sqrt{6}$ (B) 24 (C) 25 (D) 12 (E) $6\sqrt{10}$

---

13. In a box of apples, $\frac{3}{7}$ of the apples are red and the rest are green. Five more green apples are added to the box. Now $\frac{5}{8}$ of the apples are green.
How many apples are there now in the box?

(A) 32 (B) 33 (C) 38 (D) 40 (E) 48

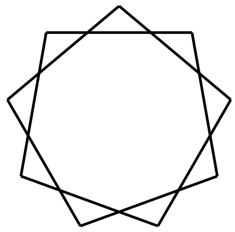
---

14. Which number exceeds its square by the greatest possible amount?

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{1}{4}$ (D) $\frac{3}{4}$ (E) $\frac{\sqrt{2}}{2}$

---

15. A regular nonagram is a nine-pointed star drawn as shown. What is the angle at each of the nine points?



(A) 100° (B) 110° (C) 120° (D) 130° (E) 140°

---

16. Two sequences are constructed, each with 900 terms:

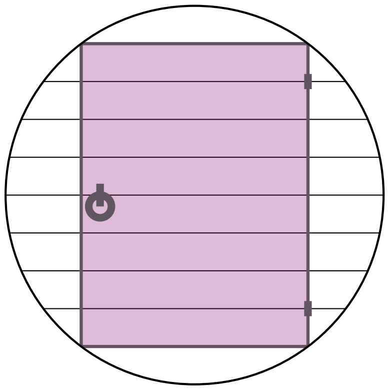
$$5, 8, 11, 14, \ldots \qquad \text{(increasing by 3)}$$
$$3, 7, 11, 15, \ldots \qquad \text{(increasing by 4)}$$

How many terms do these two sequences have in common?

(A) 400 (B) 300 (C) 275 (D) 225 (E) 75

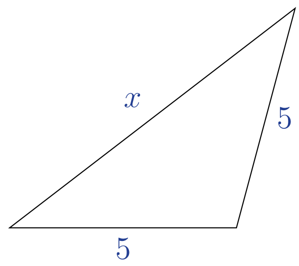
---

17. A circular steel gateway surrounding a rectangular gate is designed as shown. The total height of the gateway is divided into 10 equal intervals by equally-spaced horizontal bars.
The rectangular gate is what fraction of the area of the entire circular gateway?



(A) $\frac{48}{25\pi}$ (B) $\frac{\sqrt{3}}{\pi}$ (C) $\frac{2}{\pi}$ (D) $\frac{8\sqrt{2}}{25\pi}$ (E) $\frac{8}{5\pi}$

18. For what values of x does the triangle with side lengths 5, 5 and x have an obtuse angle?



(A) $0 < x \leq 5\sqrt{2}$ (B) $5 < x \leq 5\sqrt{2}$
(C) $5 < x < 10$ (D) $0 < x < 10$
(E) $5\sqrt{2} < x < 10$

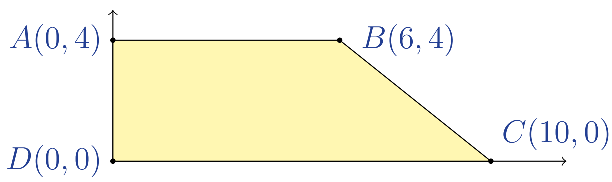
---

19. A rectangle has area 20 and perimeter 22. How long is each of its diagonals?

(A) $4\sqrt{5}$ (B) 10 (C) $\sqrt{29}$ (D) $2\sqrt{26}$ (E) 9

---

20. The line $y = mx$ divides quadrilateral $ABCD$ into two equal areas.
The value of m is



(A) 1 (B) $\frac{2}{3}$ (C) $\frac{8}{17}$ (D) $\frac{8}{15}$ (E) $\frac{8}{25}$

Questions 21 to 25, 5 marks each

21. Manny has three ways to travel the 8 kilometres from home to work: driving his car takes 12 minutes, riding his bike takes 24 minutes and walking takes 1 hour and 44 minutes. He wants to know how to get to work as quickly as possible in the event that he is riding his bike and gets a flat tyre.

He has three strategies:

(i) If he is close to home, walk back home and then drive his car.
(ii) If he is close to work, just walk the rest of the way.
(iii) For some intermediate distances, spend 20 minutes fixing the tyre and then continue riding his bike.

He knows there are two locations along the route to work where the strategy should change. How far apart are they?

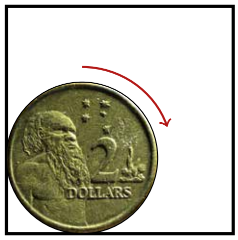
(A) 2 km (B) 3 km (C) 4 km (D) 5 km (E) 6 km

---

22. A circular coin of radius 1 cm rolls around the inside of a square without slipping, always touching the boundary of the square.

When it returns to where it started, the coin has performed exactly one whole revolution.

In centimetres, what is the side length of the square?



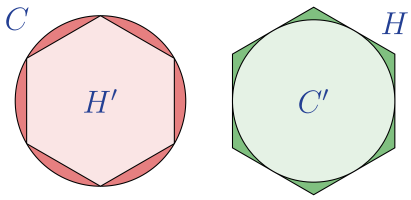
(A) π (B) 3.5 (C) $1+\pi$ (D) 4 (E) $2+\frac{\pi}{2}$

23. A passenger train 200 m long and travelling at 80 km/h passes a goods train 2 km long travelling in the opposite direction at 20 km/h. What is the distance, measured along one of the tracks, between the point at which the fronts of the trains pass each other and the point at which their back ends pass each other?

(A) 1.28 km (B) 1.4 km (C) 1.56 km (D) 1.8 km (E) 1.88 km

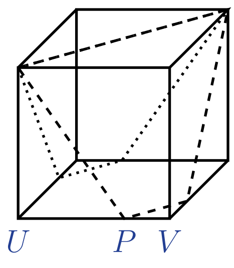
24. A circle C and a regular hexagon H have equal area. A regular hexagon H' is inscribed in C, and a circle C' is inscribed in H.

What is the ratio of the area of H' to the area of C'?



(A) $1:1$ (B) $3:\pi$ (C) $9:\pi^2$ (D) $3:4$ (E) $3\sqrt{3}:2\pi$

25. A cube of side length 1 is cut into three pieces of equal volume by two planes passing through the diagonal of the top face. One plane cuts the edge $\overline{UV}$ at the point P. What is the length PV?



(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{\sqrt{2}}{2}$ (D) $\sqrt{3}-1$ (E) $\frac{\sqrt{5}-1}{2}$

For questions 26 to 30, shade the answer as an integer from 0 to 999 in the space provided on the answer sheet.

Questions 26–30 are worth 6, 7, 8, 9 and 10 marks, respectively.

---

26. The number 35 has the property that when its digits are both increased by 2, and then multiplied, the result is $5 \times 7 = 35$, equal to the original number.
Find the sum of all two-digit numbers such that when you increase both digits by 2, and then multiply these numbers, the product is equal to the original number.

---

27. In a list of numbers, an *odd-sum triple* is a group of three numbers in a row that add to an odd number. For instance, if we write the numbers from 1 to 6 in this order,

$$6 \qquad 4 \qquad 2 \qquad 1 \qquad 3 \qquad 5$$

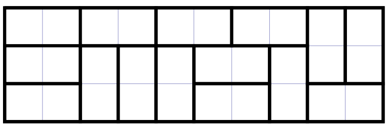
then there are exactly two odd-sum triples: $(4, 2, 1)$ and $(1, 3, 5)$.
What is the greatest number of odd-sum triples that can be made by writing the numbers from 1 to 1000 in some order?

---

28. Terry has a solid shape that has four triangular faces. Three of these faces are at right angles to each other, while the fourth face has side lengths 11, 20 and 21. What is the volume of the solid shape?

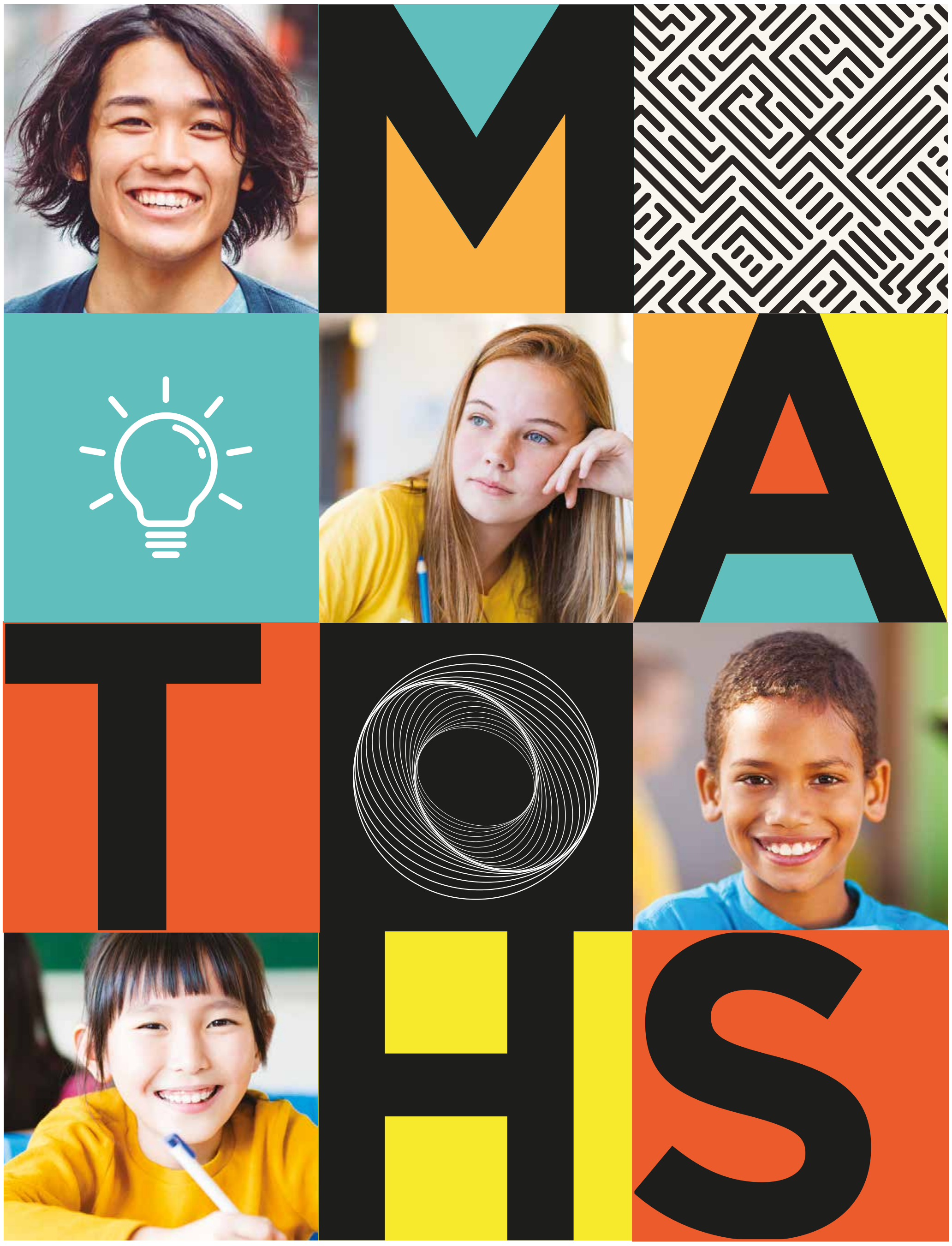
---

29. The diagram shows one way in which a 3×10 rectangle can be tiled by 15 rectangles of size 1×2.
Since this tiling has no symmetry, we count rotations and reflections of this tiling as different tilings.
How many different tilings of this 3×10 rectangle are possible?



---

30. A function f, defined on the set of positive integers, has $f(1) = 2$ and $f(2) = 3$. Also $f(f(f(n))) = n + 2$ if n is even and $f(f(f(n))) = n + 4$ if n is odd. What is $f(777)$?



SOLVE PROBLEMS. CREATE THE FUTURE.

Problems are part of life and we've made it our mission to equip young students with the skills to solve more of them. Problem solving is a life skill and by developing it, students can create more choices for themselves and the future.

AUSTRALIAN MATHS TRUST

amt.edu.au

2019 AMC — SENIOR