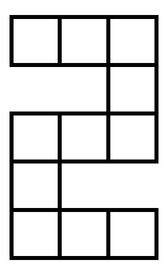
2015 World Mathematics Team Championship
Junior Level Individual Round 1

Problems	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Answers	2204	10	3	35	8	152	100	12
Problems	**9**	**10**	**11**	**12**	**13**	**14**		
Answers	30.56	8	5	36	126	3		
Problems	**15**	**16**	**17**	**18**		**19**	**20**	
Answers	$\frac{20}{3}$	374	18	47		84	1550	

1. Find the sum 19+198+1987.

Solution: 2204.
$$\begin{aligned} 19+198+1987 &= (20-1)+(200-2)+(2000-13) \\ &= 2220-(1+2+13) \\ &= 2204 \end{aligned}$$

2. As shown in the figure below, 11 identical size square cards are placed on a desk to form a figure of "2". Find the number of rectangles in this figure with an area of 2.



Solution: 10. Each 1×3 rectangle can contain two smaller rectangles of area 2. Since this figure has 5 such 1×3 rectangles, there is a total of 5×2 = 10 rectangles of area 2.

3. How many consecutive zeros are at the end of the product 25×26×27×28×29×30.

Solution: 3. 25×26×27×28×29×30 = 5×5×2×13×3×3×3×2×2×7×29×2×3×5
= (5×2) × (5×2) × (5×2) ×13×3×3×3×2×7×29×3.

It is easy to see that there are 3 consecutive zeros are at the end of this product.

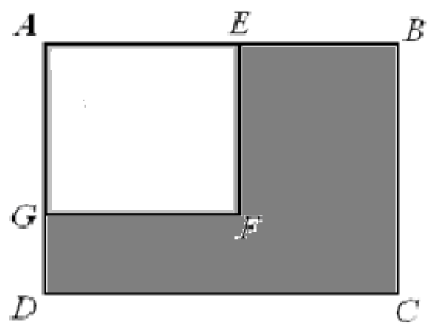
4. Suppose there 5 natural numbers with an average of 40. If 10 is one of these numbers and it is changed into another number making the new average to be 45, find this new changed number.

Solution: 35. Since the average is 40, then the sum of these 5 number is $40\times5 = 200$. However, the new average is 45, so the new sum is $45\times5 = 225$ which is 25 more than the old sum. Therefore, the number 10 has been increased also by 10 and the number is $10+25 = 35$. sum of the 5 numbers

5. Among all 2–digit numbers that are formed by choosing any two different numbers from 1, 2, 3, 4, and 5, how many are multiples of 3?

Solution: 8. Eight numbers 12, 15, 21, 24, 42, 45, 51, and 54.

6. As shown in the figure below, $ABCD$ is a rectangle and $AEFG$ is a square of side length of 10. If $EB = 8$ and $GD = 4$, find the area of the shaded portion.



Solution: 152. $AB = AE + EB = 10 + 8 = 18$. $AD = AG + GD = 10 + 4 = 14$. So, the area of the rectangle $ABCD$ is $18\times14 = 252$. Therefore the area of the shaded portion is

(Area of $ABCD$) – (Area of $AEFG$) = 252 – 100 = 152.

7. Suppose A can input 70 words per minute and B can input 74 words per minute. How long would it take for them to input a total of 14400 words?

Solution: 100. Together, A and B can input 7)+74 = 144 words. So it would take them

$14400\div144 = 100$ minutes.

8. A has four of each 10 and 5 gram weights. If she takes out one or more weights from these 8 weights, how many different total weights can she have?

Solution: 12. According to the problem, in taking out one or more weights, the smallest possible total weight is 5 grams and the largest possible total weight is 60 grams. Also, the smallest difference between any two weights is 5 grams. Therefore, there are 12 possible total weights and they are: 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, and 60 grams.

9. The figure below is composed of 5 rectangles and 4 identical sized fans with the indicated dimensions. Find the total area of shaded regions. (use 3.14 for π)

Solution: 30.56. The shaded area is $6\times3 + 3.14\times2^2 = 18 + 12.56 = 30.56$.

10. How many factors are shared by both numbers 48 and 168?

Solution: 8. The Greatest Common Factor (GCF) of 48 and 168 is 24. Hence, their common factors must also be factors of 24. Since $24 = 2\times2\times2\times3$, there are 8 common factors shared by 48 and 168 and they are: 1, 2, 3, 4, 6, 8, 12, 24.

11. Three years ago, father's age was 15 times son's age. Two years from today, father's age will be 5 times son's age. What is son's age today?

Solution: 5. Suppose the father is a years old this year and the son is b years old. Three years ago, $a–3 = 15(b–3)$ or $a = 15b – 42$ and two years from today, $a+2 = 5(b+2)$ or $a = 5b+8$. Therefore, $15b–42 = 5b+8$ or $b = 5$.

12. The figure below shows a solid that is composed by 10 cubes of edge length of 1. Find the surface area of this solid (include the bottom surface area).

Solution: 36. Each of six views (top, bottom, left, right, front, and back) of this solid is a surface that is consisted of 6 squares each of edge length of 1. Therefore, the total surface area of this solid is $6\times6\times1 = 36$.

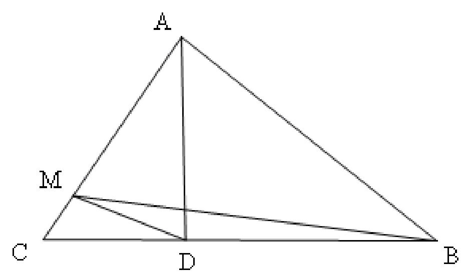
13. Let a and b be natural numbers such that $a > b$. If the Greatest Common Factor (GCF) of a and b is 42 and their Least Common Multiple (LCM) is 252, find the value of a when $(a–b)$ is smallest.

Solution: 126. We know that GCF × LCM = the product of these two numbers. Hence, $a \times b = 42 \times 252 = 42 \times 42 \times 2 \times 3$. To make $(a-b)$ smallest, a and b must be the two factors that are closest together. Therefore, $a = 42 \times 3 = 126$.

14. There are 20 people participating in an archery competition. Eighteen of them hit the target in the first round, fifteen hit the target in the second round, and only 10 people hit the target in the third round. What is the minimum possible number of people who hit the target in all three rounds?

Solution: 3. 20–18 = 2 people missed the target in the first round.
20–15 = 5 people missed the target in the second round.
20–10 = 10 people missed the target in the third round.
Therefore, there are at least 20 – (2+5+10) = 3 people hit the target in all three rounds.

15. As shown in the figure below, $BM = \frac{1}{5}AB$, $BD = \frac{1}{3}BC$, $S_{\Delta MCD} = 4$, $AD = 6$, and $AD \perp BC$. Find CD. (The figure does not drawn to scale).



Solution: $\frac{20}{3}$. Since $BD = \frac{1}{3}BC$, so $CD = \frac{2}{3}BC$ or $S_{\Delta MCD} = \frac{2}{3}S_{\Delta BMC}$. Hence, $S_{\Delta BMC} = \frac{3}{2}S_{\Delta MCD} = 6$. Also, $BM = \frac{1}{5}AB$, so $S_{\Delta BMC} = \frac{1}{5}S_{\Delta ABC}$ and $S_{\Delta ABC} = 30$. Because $AD \perp BC$ and $\mathrm{AD} = 6$, so $S_{\Delta ABC} = 30 = \frac{1}{2} \cdot BC \cdot AD = \frac{1}{2} \cdot BC \cdot 6$ which means $BC = 10$. Therefore, $CD = \frac{2}{3}BC = \frac{20}{3}$.

16. A school is holding a table tennis tournament. In the beginning, $\frac{3}{11}$ of the students in the school signed up to compete. Later, another 136 students signed up. At that time, the total number of students who signed up is $\frac{7}{4}$ times the number of students who did not sign up. How many students are in the school?

Solution: 374. The fraction of number of signed up students is $\frac{7}{4} \div (\frac{7}{4} + 1) = \frac{7}{11}$.

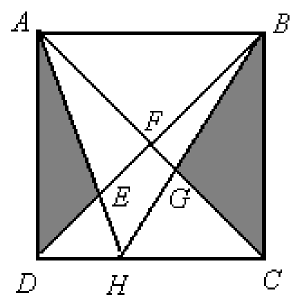
Therefore, the total number of students in this school is $136 \div (\frac{7}{11} - \frac{3}{11}) = 374$.

17. Suppose the letters W, M, T, and C represent 4 different digits and $\overline{WW} \times \overline{MM} + \overline{TW} + C = 2015$. Find the value for $W+M+T+C$.

Solution: 18. Since $\overline{WW} \times \overline{MM} + \overline{TW} + C = 2015$, $W \times M \times (11 \times 11) + \overline{TW} + C = 2015$. Also, $2015 \div (11 \times 11) = 16 \cdots\cdots 79$, so $T=7$, $W+C=9$, $W \times M = 16$. Since W, M, T, and C represent different digits, so $\begin{cases} W = 2 \\ M = 8 \end{cases}$ or $\begin{cases} W = 8 \\ M = 2 \end{cases}$.

However, when $W = 2$, C = 7 and $T = 7$ which is a contradiction. So, $W = 8$ which means $M = 2$, $T = 7$, and $C = 1$. Therefore, $W+M+T+C = 8+2+7+1 = 18$.

18. As shown in figure below, let the area of square $ABCD$ be 124 and the area of quadrilateral $EFGH$ be 16. Find the sum of the areas of the two shaded regions.



Solution: 47. Because ΔDAH and ΔDBH have the same base and same height, they have the same area. Hence,

(Area of ΔDAH) – (Area of ΔDEH) = (Area of ΔDBH) – (Area of ΔDEH) which means ΔDAE and ΔBHE have the same area. Therefore,

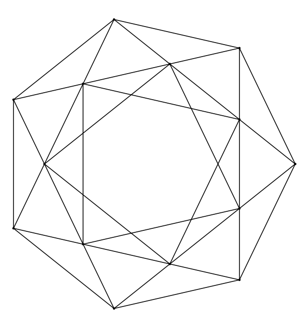
(Area of $EFGH$) = (Area of ΔBHE) – (Area of ΔFBG)

= (Area of ΔBHE + Area of ΔBCG) – (Area of ΔFBG + Area of ΔBCG)

= (Area of shaded region) – (Area of ΔBCF).

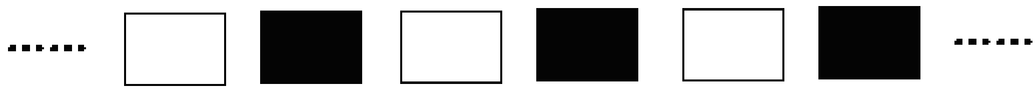
Since the area of quadrilateral $EFGH$ is 16 and the area of ΔBCF is 1/4 the area of the square, so 124÷4 = 31. Therefore, the area of the shaded region is 16+31 = 47.

19. How many triangles are in the figure below?



Solution: 84. There are 28 triangles each formed by only 1 triangle.
There are 28 triangles each formed by 2 small triangles.
There are 14 triangles each formed by 3 small triangles.
There are 14 triangles each formed by 4 small triangles.
Therefore, the total number of triangles in the figure is 28+28+14+14 = 84.

20. As shown in figures below, each of these black and white boxes contains a certain number of balls of same color as the box. Suppose each black box contains no more than 50 black balls and each white box contains a different number of white balls. If each box has at least one ball and the total number of balls in all these boxes is 2015, what is the maximum total number of black balls in all these black boxes?



Solution: 1550. In order to maximize the total number of black balls, the first box should be a black box and each black box should have the maximum number of 50 black balls. Also, to minimize the total number of white balls, only one white ball should be placed in the first white box and 2 white balls should be placed in the second white box and so on. If we consider the first black box and the first white box as group 1 and the second black box and the second white box as group 2 and so on, then the combined total of balls in group 1 is 51, the combined total in group 2 is 52 and 53 for group 3 and so on. Adding the number of balls in each group and we have $51+52+53+54+\ldots+82 = 2128$ which is larger than 2015 by $2128 - 2015 = 113$. Since $113 - 82 = 31$ and 31 is the number of white balls in white box 31, so the total number of balls up to black box 31 is exactly 2015. Therefore, the maximum number of black boxes is 31 and there are $31\times50 = 1550$ black balls.